A **Latin square** of order \( n \) is an \( n \times n \) array filled with the symbols \( \{1, \ldots, n\} \), such that no symbol is repeated twice in any row or column.

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
2 & 1 & 4 & 3 \\
3 & 4 & 1 & 2 \\
4 & 3 & 2 & 1 \\
\end{array}
\]

Similarly, a **partial Latin square** of order \( n \) is simply a Latin square where we allow ourselves to leave some cells blank

\[
\begin{array}{c|c|c|c}
1 & & & \\
\hline
 & 1 & & \\
 & & 1 & \\
 & & & 2 \\
\end{array}
\]

We say that a partial Latin square is **completable** if there is some way to fill in its blank cells so that we get a Latin square. The task of determining whether a given partial Latin square is completable, and if so finding its completion, is one of the more popular puzzles in recreational mathematics — **Sudoku grids**, for example, are just special cases of \( 9 \times 9 \) Latin squares!

Despite this rather silly-sounding description, the study of Latin squares is a remarkably rich and complicated area of mathematics. Latin squares and their completions have applications to fields in error-correcting codes, finite geometry, efficient router design and cryptography, while themselves providing a large number of mathematically interesting and yet unsolved questions in mathematics.

This project is into one such set of questions. Specifically, we seek to study with this project the collection of **\( \epsilon \)-sparse partial Latin squares**: those partial Latin squares that have no more than \( \epsilon n \) nonblank entries in any given row or column, and furthermore contain no more than \( \epsilon n \) many copies of any one symbol through the entire square.

A famous conjecture of Daykin and Häggkvist claims that all \( \frac{1}{4} \)-dense partial Latin squares are completable. At the same time, a famous result of Smetianuk proves that any arbitrary Latin square containing no more than \( n - 1 \) filled cells is completable.

This project will explore the squares “in between” these two claims. In particular, suppose that we have a square that is \( \epsilon \)-dense, for some fixed value of \( \epsilon \). We know that when \( \epsilon = 1 \), then any square containing at most \( n - 1 \) filled cells is completable. Conversely, when \( \epsilon = \frac{1}{4} \), then Daykin and Häggkvist’s conjecture states that any square is completable.

We seek to study what happens for values of \( \epsilon \) between \( \frac{1}{4} \) and 1. For example, suppose we have a \( \frac{1}{4} \)-dense partial Latin square \( P \), containing \( k \) cells. For what values of \( k \) is this square completable? We seek to bound and ideally determine this value of \( k \) for as many values of \( \epsilon \) as possible. With luck, work here will shed light on how we might proceed in a proof of Daykin and Häggkvist’s original conjecture on \( \frac{1}{4} \)-dense partial Latin squares itself!