Numerical properties of linearizations of matrix polynomials expressed in the Chebyshev basis.
Advisor: Maria Isabel Bueno Cachadina

Before I explain the background of the problem we will work on, let me mention some important information:

- The only math background that you need to work on the project that I am proposing is upper-division Linear Algebra. Some knowledge of Numerical Analysis could be helpful.
- Being capable of writing good proofs is a must.
- Having programming skills in Matlab is desirable. If you don't know how to program in Matlab, no worries. We will teach you here the basics.

When you read the description of the project below, you may feel overwhelmed by lots of technical words that you have not heard before. Do not worry. We will spend some time learning the background in depth before we start working on the project. So, if you like Linear Algebra, this is your project!

Also note that my goal is to publish papers with you, in refereed journals if possible. You can check my webpage

`math.ucsb.edu/~mbueno/publications.html`

to see the papers that I have published or submitted with undergraduate students so far.

Description of the project.

Let

$$P(\lambda) = A_k\lambda^k + A_{k-1}\lambda^{k-1} + \cdots + A_0 \tag{1}$$

be a matrix polynomial of degree $k \geq 2$, where the coefficients A_i are $n \times n$ matrices with entries in the field $\mathbb{C}$ of complex numbers. In particular, if $k = 1$, $P(\lambda)$ is called a pencil. Matrix polynomials arise in many applications such like systems of differential-algebraic equations, vibration analysis of structural systems, acoustics, fluid-structure interaction problems, computer graphics, signal processing, control theory, and linear system theory.

A matrix pencil $L_P(\lambda) = \lambda L_1 - L_0$, with $L_1, L_0 \in M_{kn}(\mathbb{C})$, is a *linearization* of $P(\lambda)$ if there exist two unimodular matrix polynomials (i.e. matrix polynomials with constant nonzero determinant), $U(\lambda)$ and $V(\lambda)$, such that

$$U(\lambda)L(\lambda)V(\lambda) = \begin{bmatrix} I_{(k-1)n} & 0 \\ 0 & P(\lambda) \end{bmatrix}.$$

Here $I_{(k-1)n}$ denotes the $(k-1)n \times (k-1)n$ identity matrix.

Beside other applications, linearizations of matrix polynomials are used in the study of the polynomial eigenvalue problem, that is, in the search of scalars λ_0 and nonzero vectors x such that $P(\lambda_0)x = 0$. In the classical approach to this problem the original matrix polynomial $P(\lambda)$ is replaced by a linearization of $P(\lambda)$. Then, standard methods for linear eigenvalue problems are applied.

The *reversal* of the matrix polynomial $P(\lambda)$ in (1) is the matrix polynomial obtained by reversing the order of the matrix coefficients of $P(\lambda)$, that is,

$$rev(P(\lambda)) := \sum_{i=0}^{k} \lambda^i A_{k-i}.$$

A linearization $L_P(\lambda)$ of a matrix polynomial $P(\lambda)$ is called a *strong linearization* of $P(\lambda)$ if $rev(L_P(\lambda))$ is a linearization of $rev(P(\lambda))$. Strong linearizations of $P(\lambda)$ have the same finite and infinite elementary divisors as $P(\lambda)$, therefore they preserve not only the finite but the infinite eigenvalues. Moreover, if $P(\lambda)$ is regular (i.e. $det(P(\lambda)) \not\equiv 0$), any linearization with the same infinite elementary divisors as $P(\lambda)$ is a strong linearization.

In the literature, many papers have been published in the last decade in which families of linearizations have been constructed for matrix polynomials. The next step should be to determine the numerical properties of these

linearizations in terms of eigenvalue conditioning and backward error so that the user knows how to choose an appropriate linearization. Several papers have been published recently in this regard. In many applications though, in the solution of nonlinear eigenvalue problems, a matrix polynomial approximation is constructed to replace the nonlinear eigenvalue problem via a Chebyshev interpolation. Then, the nonlinear eigenvalue problem is replaced by a polynomial eigenvalue problem but the resulting matrix polynomial is expressed in terms of the Chebyshev basis instead of the monomial basis.

In summer 2019, we will study the numerical properties of linearizations of matrix polynomials expressed in the Chebyshev basis. The Chebyshev basis can be replaced, in this context, by other polynomial bases and study the equivalent problem.