Derivative Practice

First, a table of the rules we have learned (does not include implicit and logarithmic techniques).

$f(x)$	$f'(x)$	Basic Example	Practice
c	0	$\frac{d}{dx}[5] = 0$	$\frac{d}{dx}\left[e^{3\sin\sqrt{3}}\right] =$
x	1		
x^k	kx^{k-1}	$\frac{d}{dx}\left[x^{10}\right] = 10x^9$	$\frac{d}{dx}\left[\frac{1}{x^2}\right] =$ $\frac{d}{dx}\left[\sqrt{x}\right] =$ $\frac{d}{dx}\left[x^e\right] =$
e^x	e^x		
$\ln x$	$\frac{1}{x}$		
Trig	See Below	$\frac{d}{dx}[\sin x] = \cos x$	$\frac{d}{dx}[\csc x] =$
Inverse Trig	See Below	$\frac{d}{dx}[\arctan x] = \frac{1}{x^2+1}$	$\frac{d}{dx}[\arccos x] =$
$cg(x)$	$cg'(x)$	$\frac{d}{dx}\left[5x^3\right] = 15x^2$	$\frac{d}{dx}[7\ln x] =$
$g(x) + h(h)$	$g'(x) + h'(x)$	$\frac{d}{dx}\left[e^x + x^6\right] = e^x + 6x^5$	$\frac{d}{dx}\left[6(\tan x - 4\sqrt[3]{x})\right] =$
$g(x)h(x)$	$g(x)h'(x) + h(x)g'(x)$	$\frac{d}{dx}\left[x^2\cos x\right] =$ $x^2(-\sin x) + (\cos x)(2x)$	$\frac{d}{dx}\left[e^x(\ln x - \frac{2}{x})\right] =$ $\frac{d}{dx}\left[(-4x^2+1)(\sec x + 4)\right] =$ $\frac{d}{dx}\left[x^3 e^x \sin x\right] =$
$\frac{g(x)}{h(x)}$	$\frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$	$\frac{d}{dx}\left[\frac{\ln x}{\cos x}\right] = \frac{\cos x(1/x) - \ln x(-\sin x)}{\cos^2 x}$	$\frac{d}{dx}\left[\frac{\sqrt{x}-4}{4x^3-5x-1}\right] =$
$g(h(x))$	$h'(x)g'(h(x))$	$\frac{d}{dx}\left[\sin(x^5)\right] = 5x^4\cos(x^5)$	$\frac{d}{dx}\left[\sqrt{\tan x - 5}\right] =$

Here are the trig and inverse trig formulas. You must know the trig ones. The inverse trig ones will be important in 3B.

$\frac{d}{dx}[\sin x] = \cos x$ $\qquad$ $\frac{d}{dx}[\tan x] = \sec^2 x$ $\qquad$ $\frac{d}{dx}[\sec x] = \sec x \tan x$

$\frac{d}{dx}[\cos x] = -\sin x$ $\qquad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\qquad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}[\arctan x] = \frac{1}{x^2+1}$ $\qquad$ $\frac{d}{dx}[\text{arcsec}\, x] = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx}[\arccos x] = -\frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}[\text{arccot}\, x] = -\frac{1}{x^2+1}$ $\qquad$ $\frac{d}{dx}[\text{arccsc}\, x] = -\frac{1}{x\sqrt{x^2-1}}$

For an *implicit* equation (one which has not been solved for y), we take the derivative of both sides bearing in mind the rule $\frac{d}{dx}\left[g(y)\right] = y'g'(y)$. At the end, solve for y'. Here is an example worked out:

$$\begin{aligned} y - \sin x &= \cos y + xy && \text{take deriv of both sides} \\ y' - \cos x &= y'(-\sin y) + (xy' + y) && \text{solve for } y' \\ y' + y'\sin y - xy' &= y + \cos x \\ y' &= \frac{y + \cos x}{1 + \sin y - x} \end{aligned}$$

Practice: Find y' if $\tan(xy) = y^3 + x\sqrt{y}$.

Some functions are of the form $g(x)^{h(x)}$, where both the base and the exponent are functions of x. This is where you need logarithmic differentiation. Here is an example worked out:

$$\begin{aligned} f(x) &= (\ln x)^{x^2-1} && \text{it's easier if you use } y \text{ instead of } f(x) \\ y &= (\ln x)^{x^2-1} && \text{take ln of both sides} \\ \ln y &= \ln((\ln x)^{x^2-1}) && \text{use log rule} \\ \ln y &= (x^2-1)\ln(\ln x) && \text{take the deriv of both sides} \\ y'\frac{1}{y} &= (x^2-1)\left(\frac{1}{x}\right)\left(\frac{1}{\ln x}\right) + \ln(\ln x)(2x) && \text{solve for } y' \\ y' &= y\left[(x^2-1)\left(\frac{1}{x}\right)\left(\frac{1}{\ln x}\right) + \ln(\ln x)(2x)\right] && \text{substitute for } y \text{ to make it a function of } x \\ f'(x) &= (\ln x)^{x^2-1}\left[(x^2-1)\left(\frac{1}{x}\right)\left(\frac{1}{\ln x}\right) + \ln(\ln x)(2x)\right] \end{aligned}$$

Practice: Find the derivative of $(\cos x)^{\sqrt{x}}$.