Curve Sketching Practice

To start with recall the four features I look for in your sketch: domain, asymptotes, first derivative information, second derivative information. It will help you to record this information as you go. In **bold** are the general steps to follow. After that are hints for this problem in particular.

Example 1: $f(x) = x^4 - 4x^3 + 1$

a) Domain:
b) Vertical Asymptotes:
c) Horizontal Asymptotes:
d) Increasing:
e) Decreasing:
f) Local mins:
g) Local maxs:
h) Other critical points:
i) Concave up:
j) Concave down:
k) Inflection points:

a) **Find the domain of the function. It's usually easiest to think "what can go wrong;" e.g. dividing by 0, square root of a negative, logarithm of a non-positive, etc.** (This one is easy)

b) **Test for vertical asymptotes at the boundaries of the domain. That is, wherever the function starts or stops being defined. At such points, take the limit from the + and − sides.** Since the function is a polynomial, it is always continuous, and so it can't have any vertical asymptotes.

c) **For horizontal asymptotes, we take $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.** We haven't worked on this much, so I won't be emphasizing it on the test. However, in this case, we can do it by writing

$$\lim_{x\to\infty} x^4 - 4x^3 + 1 = \lim_{x\to\infty} x^4(1 - 1/x + 1/x^4) = \lim_{x\to\infty} x^4(1) = \infty$$

$$\lim_{x\to-\infty} x^4 - 4x^3 + 1 = \lim_{x\to-\infty} x^4(1 - 1/x + 1/x^4) = \lim_{x\to-\infty} x^4(1) = \infty$$

So there are no horizontal asymptotes, but we can see the "end behavior" is that the function goes to ∞ on both ends.

d-h) **Take the first derivative**

Find the critical points. Remember, critical points are x values where either the derivative doesn't exist or where it is zero. You should check for both. In this case, there are two critical points to find.

Draw a number line with the critical points labeled.

Use test points in between and on the outside of your critical points to determine where the function is increasing and decreasing In this case, there are three places to check. For example, your point to the left might be -10. If you get a positive number, write "increasing" and/or draw an arrow with positive slope. If you get a negative number, write "decreasing" and/or draw an arrow with negative slope. Fill in the increasing/decreasing intervals above.

From this information, you should be able to see which critical points are local mins, local maxs, and which are neither. Find the y values for these points. Fill in the information above.

i-k) **Take the second derivative. Test for possible inflection points by setting it equal to zero (or seeing where it doesn't exist). This is the same as finding the critical points of the first derivative.**

Again, draw a number line, and test points in between your possible inflection points to determine concavity. If the graph switches from concave up to down or down to up, that is an inflection point.

Now, use this information to sketch the graph. Start by plotting the points you know (i.e. the critical points and inflection points). Next, you would be drawing asymptotes, if there were any. Now, connect the dots making sure your function is increasing/decreasing in the right places and concave up/down in the right places. It doesn't have to be

to scale, but you should label your points, and they should be in the right relative positions. After you're done, you can type the function into your graphing calculator or wolframalpha to see if you got the right idea.

Example 2: $f(x) = xe^x$

a) Domain:
b) Vertical Asymptotes:
c) Horizontal Asymptotes:
d) Increasing:
e) Decreasing:
f) Local mins:
g) Local maxs:
h) Other critical points:
i) Concave up:
j) Concave down:
k) Inflection points:

See if you can follow the steps in bold above to arrive at the answers here and a suitable graph. The one hint I'll give is that there is a horizontal asymptote you must find using l'Hospital's rule.

Example 3: $f(x) = \frac{x-1}{x^2}$

a) Domain:
b) Vertical Asymptotes:
c) Horizontal Asymptotes:
d) Increasing:
e) Decreasing:
f) Local mins:
g) Local maxs:
h) Other critical points:
i) Concave up:
j) Concave down:
k) Inflection points:

See if you can follow the steps in bold above to arrive at the answers here and a suitable graph. In this graph there is a horizontal asymptote and a vertical asymptote.