Math 34B Spring Quarter Midterm Examination  
May 4, 2006

NAME: \underline{Answer Key (\checkmark).} 

TA & DISCUSSION SECTION: 

You have 70 minutes in which to complete this examination. Attempt all of the questions. Note that you will not be awarded full credit on a question unless your answer is clearly, carefully and neatly stated.

UP TO 5 BONUS POINTS (ADDED DIRECTLY TO YOUR SCORE ON THE EXAMINATION) WILL BE AWARDED FOR NEATLY AND CAREFULLY PRESENTED WORK!

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<thead>
<tr>
<th>Problem</th>
<th>Maximum Score</th>
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(1) Find the derivatives with respect to \( x \) of the following functions:

(i) \( f(x) = 4 \sin(2x) + 2x + 6 \)

\[
\begin{align*}
\frac{d}{dx} f(x) &= 2 \cdot 4 \cos(2x) + 2x + 6 \\
&= 8 \cos(2x) + 2x + 6
\end{align*}
\]

(ii) \( f(x) = e^{2x} \sin(4x) \)

\[
\begin{align*}
\frac{d}{dx} f(x) &= \frac{d}{dx} (e^{2x}) \sin(4x) + e^{2x} \frac{d}{dx} (\sin(4x)) \\
&= 2e^{2x} \sin(4x) + 4e^{2x} \cos(4x) \\
&= 2e^{2x} \left( \sin(4x) + 2 \cos(4x) \right).
\end{align*}
\]

(iii) \( y = (f(x))^2 \)

\[
\begin{align*}
y &= f(x) \cdot f(x), \\
\frac{dy}{dx} &= \frac{d}{dx} (f(x) \cdot f(x)) = f'(x) \cdot f(x) + f(x) \cdot f'(x) \\
&= 2f(x)f'(x).
\end{align*}
\]
(2) Find the following integrals:

(i) \( \int 6e^{2t} \, dt \)

\[
\int 6e^{2t} \, dt = 3e^{2t} + C
\]

(ii) \( \int_2^4 x^2 \, dx \)

\[
\int_2^4 x^2 \, dx = \left[ \frac{x^3}{3} \right]_2^4 = \frac{4^3}{3} - \frac{2^3}{3} = \frac{56}{3}
\]

(iii) \( \int_1^\pi 2 \, dx \)

\[
\int_1^\pi 2 \, dx = \left[ 2x \right]_1^\pi = 2\pi - 2.
\]
(3) Consider the function \( f(t) = 4 \cos(2t + 2) \), where \( t \) denotes time measured in seconds.

(a) What is the period of \( f(t) \)?

The period of \( f(t) \) is \( \frac{\pi}{2} \) seconds.

(b) What is the frequency of \( f(t) \)?

The frequency of \( f(t) \) is \( \frac{1}{\pi} \) Hz.

(c) What is the amplitude of \( f(t) \)?

The amplitude of \( f(t) \) is equal to 4.
(4) The height above the ground in metres of a rocket \( t \) seconds after being launched is 
\[ h(t) = 10t + 2t^2. \]
(a) What is the velocity of the rocket after \( t \) seconds?

Let \( v(t) = \text{velocity of the rocket after } t \text{ seconds}. \)

Then
\[
\begin{align*}
v(t) &= \frac{dh}{dt} \\
&= \frac{d}{dt} (10t + 2t^2) \\
&= 10 + 4t \text{ m/s.}
\end{align*}
\]

(b) How many seconds after the launch is the velocity of the rocket equal to \( 50 \text{ m/s} \)?

The velocity of the rocket is \( 50 \text{ m/s} \) when
\[
10 + 4t = 50
\]
\[
\Rightarrow 4t = 50 - 10 \\
\Rightarrow 4t = 40 \\
\Rightarrow t = 10 \text{ seconds.}
\]

Hence the velocity of the rocket is equal to \( 50 \text{ m/s} \) after \( 10 \) seconds.

(c) What is the acceleration of the rocket \( t \) seconds after being launched?

Let \( a(t) = \text{acceleration of the rocket after } t \text{ seconds}. \)

Then
\[
\begin{align*}
a(t) &= \frac{dv}{dt} \\
&= \frac{d}{dt} (10 + 4t) \\
&= 4 \text{ m/s}^2.
\end{align*}
\]
(5) The radius of a circular oil slick is increasing at a rate of 1 metre per second. At time \( t = 0 \), the area of the slick is equal to \( 4\pi \text{ m}^2 \).

(a) Find the radius of the slick after \( t \) seconds

Let \( r(t) = \) radius of oil slick after \( t \) seconds.

Then \( \frac{dr}{dt} = 1 \), and so \( r(t) = \int 1 \cdot dt = t + C \).

When \( t = 0 \), the radius of the slick is 2 m (since its area is \( 4\pi \text{ m}^2 \)). So

\[ 2 = 0 + C, \]

and therefore \( C = 0 \).

\[ r(t) = t + 2 \text{ metres}. \]

(b) Find the rate at which the area of the slick is increasing after \( t \) seconds.

Let \( A(t) = \) area of the slick after \( t \) seconds.

Then \( A(t) = \pi r(t)^2 \)

\[ = \pi (t + 2)^2 \]

\[ \therefore \frac{dA}{dt} = 2\pi (t + 2) \text{ m}^2/\text{s}. \]

Hence the area of the slick is increasing at a rate of \( 2\pi (t + 2) \text{ m}^2/\text{s} \) after \( t \) seconds.