Math 34B Spring Quarter Midterm Examination
May 4, 2006

NAME: __________________________

Answer Key: __________

TA & DISCUSSION SECTION:

You have 70 minutes in which to complete this examination. Attempt all of the questions. Note that you will not be awarded full credit on a question unless your answer is clearly, carefully and neatly stated.

UP TO 5 BONUS POINTS (ADDED DIRECTLY TO YOUR SCORE ON THE EXAMINATION) WILL BE AWARDED FOR NEATLY AND CAREFULLY PRESENTED WORK!

<table>
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<th>Problem</th>
<th>Maximum Score</th>
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<td>NEATNESS BONUS</td>
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(1) Find the derivatives with respect to $x$ of the following functions:

(i) $f(x) = 9 \sin(3x) + 3x + 1$

\[ f'(x) = 27 \cos(3x) + 3 \]

(ii) $f(x) = e^{3x} \sin(5x)$

\[ f'(x) = \frac{d}{dx} \left( e^{3x} \right) \sin(5x) + e^{3x} \frac{d}{dx} \left( \sin(5x) \right) \]
\[ = 3e^{3x} \sin(5x) + 5e^{3x} \cos(5x) \]
\[ = e^{3x} \left( 3\sin(5x) + 5\cos(5x) \right) \]

(iii) $y = (g(x))^2$

\[ y = g(x) \cdot g(x) \]

\[ \therefore \quad \frac{dy}{dx} = g'(x) \cdot g(x) + g(x) \cdot g'(x) \]
\[ = 2g(x) \cdot g'(x). \]
(2) Find the following integrals:
(i) \( \int 3e^{at} \, dt \)

\[
\int 3e^{at} \, dt = \frac{1}{3} e^{at} + C
\]

(ii) \( \int_1^3 x^3 \, dx \)

\[
\int_1^3 x^3 \, dx = \left[ \frac{x^4}{4} \right]_1^3
\]

\[
= \frac{3^4}{4} - \frac{1^4}{4}
\]

\[
= \frac{81}{4} - \frac{1}{4}
\]

\[
= \frac{80}{4}
\]

\[
= 20
\]

(iii) \( \int_1^\pi 3 \, dx \)

\[
\int_1^\pi 3 \, dx = \left[ 3x \right]_1^\pi
\]

\[
= 3\pi - 3.
\]
(3) Consider the function $f(t) = 5 \cos(3t + 3)$, where $t$ denotes time measured in seconds.

(a) What is the period of $f(t)$?

The period of $f(t)$ is $\frac{2\pi}{3}$ seconds.

(b) What is the frequency of $f(t)$?

The frequency of $f(t)$ is $\frac{3}{2\pi}$ Hertz.

(c) What is the amplitude of $f(t)$?

The amplitude of $f(t)$ is equal to 5.
(4) The height above the ground in metres of a guided missile \( t \) seconds after being launched is \( h(t) = 9t + 3t^2 \).

(a) What is the velocity of the missile after \( t \) seconds?

Let \( v(t) \) = Velocity of the missile after \( t \) seconds.

Then

\[
v(t) = \frac{dh}{dt} = \frac{d}{dt} (9t + 3t^2)
= 9 + 6t \text{ m/s}.
\]

(b) How many seconds after the launch is the velocity of the missile equal to \( 45 \text{ m/s} \)?

The velocity of the missile is \( 45 \text{ m/s} \) when

\[
9 + 6t = 45
\]

\[
\therefore \ 6t = 45 - 9
= 36
\]

\[
\therefore \ t = 6 \text{ seconds}.
\]

Hence the velocity of the missile is equal to \( 45 \text{ m/s} \) after 6 seconds.

(c) What is the acceleration of the missile \( t \) seconds after being launched?

Let \( a(t) \) = acceleration of the missile after \( t \) seconds.

Then

\[
a(t) = \frac{dv}{dt}
= \frac{d}{dt} (9 + 6t)
= 6 \text{ m/s}^2.
\]
(5) The radius of a circular tide pool is increasing at a rate of 1 metre per second. At time \( t = 0 \), the area of the tide pool is equal to \( 9\pi \text{ m}^2 \).

(a) Find the radius of the tide pool after \( t \) seconds.

Let \( r(t) \) = radius of tide pool after \( t \) seconds

Then \( \frac{dr}{dt} = 1 \), and so \( r(t) = \int 1 \cdot dt = t + C \)

When \( t = 0 \), the radius of the tide pool is 3 m (since its area is \( 9\pi \text{ m}^2 \)). So

\[ 3 = 0 + C, \]

and therefore \( C = 3 \).

\( \therefore \) \( r(t) = t + 3 \) metres.

(b) Find the rate at which the area of the tide pool is increasing after \( t \) seconds.

Let \( A(t) \) = area of the tide pool after \( t \) seconds.

Then \( A(t) = \pi \cdot r(t)^2 \)

\[ = \pi \cdot (t + 3)^2 \]

\[ \therefore \frac{dA}{dt} = 2\pi (t + 3) \text{ m}^2/\text{s} \]

Hence the area of the tide pool is increasing at a rate of \( 2\pi (t+3) \text{ m}^2/\text{s} \) after \( t \) seconds.