1. Which of the following numbers are algebraic integers:
   (i) $\sqrt[3]{15}(\sqrt[7]{7} + \sqrt[3]{39})$
   (ii) $\frac{1+i}{\sqrt{2}}$
   (iii) $\frac{1+i}{3} \sqrt[3]{10} + \sqrt[3]{100}$

2. (i) Let $d$ be a squarefree integer. Find the ring of integers of $\mathbb{Q}(\sqrt{d})$.
   (ii) Let $d$ be a squarefree integer with the property that $d \equiv 1 \pmod{4}$. Show that $\mathbb{Z}(\sqrt{d})$ is not a PID.

3. Let $L/K$ be a finite, separable extension of fields (not necessarily of characteristic 0).
   (i) Show that $Tr_{L/K} : L \times L \to K; (x, y) \mapsto Tr_{L/K}(xy)$ is a non-degenerate, symmetric, $K$-bilinear form on $L$.
   (ii) Show that the map $Tr_{L/K} : L \to K; x \mapsto Tr_{L/K}(x)$ is surjective.
   (Hint for both (i) and (ii): Artin’s theorem on linear independence of characters.)

4. Suppose that $K$ is a number field, and let $x \in \mathfrak{O}_K$. Show that $x$ is a unit in $\mathfrak{O}_K$ if and only if $N_{K/\mathbb{Q}}(x) = \pm 1$.
   (Hint: You may find it helpful to view $K$ as a $\mathbb{Q}$-vector space and then consider the characteristic equation of the “multiplication by $x$” map on $K$.)

5. Let $\zeta^n = 1$ and assume that $\alpha = \frac{1}{m}(\sum_{i=1}^{m} \zeta^{\kappa_i})$ is an algebraic integer. Show that either $\sum_{i=1}^{m} \zeta^{\kappa_i} = 0$ or $\zeta^{\kappa_1} = \zeta^{\kappa_2} = \ldots = \zeta^{\kappa_m}$.
   (Hint: Set $K = \mathbb{Q}(\zeta)$. First consider $N_{K/\mathbb{Q}}(\alpha)$, and see what this tells you about $\alpha$. Then think geometrically.)