Math 3B Midterm 1 Review

1. Estimate the area under the graph of \( e^{-x^2} \) from \(-2 \leq x \leq 2\) using 4 rectangles; and using left, right and midpoints.

2. The speed of a runner increases steadily the first three seconds of a race. Her speed at half second intervals is given in the table. Find lower and upper estimates for the distance that she travelled during those three seconds.

<table>
<thead>
<tr>
<th>( t(s) )</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v(\text{ft/s}) )</td>
<td>0</td>
<td>6.2</td>
<td>10.8</td>
<td>14.9</td>
<td>18.1</td>
<td>19.4</td>
<td>20.2</td>
</tr>
</tbody>
</table>

3. Find the Riemann sum for \( f(x) = x - 2\sin(2x) \) for \( 0 \leq x \leq 3 \) with six terms, taking the sample points to be right endpoints. Explain what the sum represents with a sketch.

4. Find the integral using the definition (limits and Riemann sums)

(a) \( \int_{-1}^{3}(1+3x)\,dx \)

(b) \( \int_{1}^{5}(2+3x-x^2)\,dx \)

(c) \( \int_{0}^{3}(1+2x^3)\,dx \)

5. Use areas to evaluate the integral

(a) \( \int_{-3}^{0}(1+\sqrt{9-x^2})\,dx \)

(b) \( \int_{-3}^{3}|3x-5|\,dx \)

6. (a) If \( w'(t) \) is the rate of growth of a child in pounds per year, what does \( \int_{5}^{10}w'(t)\,dt \) represent?

(b) The current in a wire is defined as the derivative of the charge \( I(t) = Q'(t) \). What does \( \int_{a}^{b}I(t)\,dt \) represent?

(c) A honeybee population starts with 100 bees and increases at a rate of \( n'(t) \) bees per week. What does \( 100 + \int_{0}^{5}5n'(t)\,dt \) represent?

7. Evaluate the integral

(a) \( \int_{1}^{2}\frac{1}{x^2}\,dx \)

(b) \( \int_{0}^{2\pi}\cos \theta\,d\theta \)

(c) \( \int_{1}^{2}\frac{4+u^2}{u^3}\,du \)

(d) \( \int_{1}^{8}\frac{x-1}{\sqrt{x^3}}\,dx \)

(e) \( \int_{0}^{\frac{\pi}{4}}\frac{1+\cos^2 \theta}{\cos \theta}\,d\theta \)

(f) \( \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\sin 3x}{1+x^4}\,dx \) (hint: use symmetry)
8. Find the general indefinite integral

(a) \( \int x \sqrt{x} \, dx \)
(b) \( \int \frac{6}{1 + x^2} \, dx \)
(c) \( \int - \csc x \cot x \, dx \)
(d) \( \int 2e^x + 4 \cos x \, dx \)

9. The velocity of a particle is \( v(t) = 3t - 5 \) meters per second, \( 0 \leq t \leq 3 \). Find the displacement and distance traveled.

10. Let \( g(x) = \int_0^x f(t) \, dt \) where \( f \) is given as the piecewise function 
\[ f(t) = \begin{cases} \sqrt{1 - t^2} & \text{if } 0 \leq t \leq 1 \\ t - 1 & \text{if } 1 \leq t \leq 3 \\ -t + 5 & \text{if } 3 \leq t \leq 7 \end{cases} \]

(a) Graph the function \( f \).
(b) Find \( g(x) \) for \( x = 0, 1, 2, 3, \ldots, 7 \).
(c) Graph \( g \). Where does \( g \) have a maximum? Where is \( g \) increasing?

11. Use part 1 of FTC to find the derivative

(a) \( g(x) = \int_0^x \sqrt{1 + 2t} \, dt \)
(b) \( g(x) = \int_1^x \ln t \, dt \)
(c) \( F(x) = \int_x^{10} \tan \theta \, d\theta \)
(d) \( y = \int_3^{5x} \frac{\cos t}{t} \, dt \)
(e) \( \int_{\cos x}^{\tan x} \cos^2 u \, du \)
(f) \( \int_{e^{3x}}^{e^x} \frac{1}{2 \ln t} \, dt \)

12. Evaluate the integral

(a) \( \int \frac{1 + 4x}{\sqrt{1 + x + 2x^2}} \, dx \)
(b) \( \int e^x (1 + e^x)^{3/2} \, dx \)
(c) \( \int \sin^3 x \cos x \, dx \)
(d) \( \int_0^\frac{\pi}{2} e^{\sin x} \cos x \, dx \)
(e) \( \int_0^1 \frac{\sin^{-1} x}{\sqrt{1 - x^2}} \, dx \)
(f) \( \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2}{1 + x^6} \, dx \)