Math 108A - Home Work # 1

1. Write the following complex numbers in the form $x + yi$ for $x, y \in \mathbb{R}$.

   (a) $\dfrac{2 + i}{1 - 3i}$;   (b) $e^{2+\pi i/3}$;   (c) $(1 + i)^8$;   (d) $\dfrac{1}{a + bi}$, $(a, b \in \mathbb{R})$.

2. For any $z \in \mathbb{C}$, prove that $z \in \mathbb{R}$ if and only if $\bar{z} = z$.

3. Verify that the subset $\{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\} \subseteq \mathbb{R}^3$ is a vector space (with the usual vector addition and scalar multiplication).

4. Let V be a vector space over F. Using only the vector space axioms, show that for any $v \in V$, the additive inverse of v is given by $-1 \cdot v$. Mention which axiom you are using in each step of the proof.

5. Let V be a vector space over F. Show that $-(-v) = v$ for any $v \in V$.