
2. Give an example of two vector spaces $V$ and $W$ and two linear maps $T : V \to W$ and $S : W \to V$ such that $ST = I_V$ but $TS \neq I_W$. In your example, is either of $S, T$ injective? Is either surjective?

3. Let $T : V \to W$ be a linear map, and let $\{v_1, \ldots, v_n\}$ be a basis for $V$. Show that $T$ is invertible if and only if $\{Tv_1, \ldots, Tv_n\}$ is a basis for $W$. (You can use questions from the previous homework (eg., 5 and 7 on p. 59) to shorten your argument.)

4. Let $A$ be an $n \times n$ matrix with entries in $F$.
   
   (a) Show that $A$ is invertible if and only if its columns are linearly independent (column) vectors in $F^n$. (Since $A$ has $n$ columns and $n = \dim F^n$, we could also say that $A$ is invertible if and only if its columns are a basis of $F^n$.) Hint: this is a consequence of the previous exercise.

   (b) Show that $A$ is invertible if and only if its rows are linearly independent vectors in $F^n$. (Here, it might be easier to replace “$A$ is invertible” with “$A$ is surjective” and note why these are equivalent.)