# Math 108A - Midterm 1 Redux

Due: Friday, May 2, 2008

**Instructions.**

- This is an optional homework. Its score will count towards your first midterm only. Basically, completing this assignment will allow you to bring a nonpassing grade up to a passing grade, or a passing grade up at most 1 letter grade.
- Problems 1-3 are from the midterm. You may redo any of these problems (or just the parts you missed) on which you received no more than two-thirds the total possible score (ie., $\leq 6$ on 1, $\leq 7$ on 2, $\leq 6$ on 3). However, they will be graded out of fewer points this time.
- Problems 4-6 are additional problems on the same material. A small number of additonal points will be added to your midterm score for each one you complete.
- As this is a continuation of your midterm, you should work independently. However, you may use the book or your notes.
- Please turn this in with your midterm.

1. Recall $\mathcal{P}(\mathbb{R})$ is the $\mathbb{R}$-vector space of polynomials in $x$ with coefficients in $\mathbb{R}$. Which of the following are subspaces of $\mathcal{P}(\mathbb{R})$? Justify your answers.
   (a) $U = \{p(x) \in \mathcal{P}(\mathbb{R}) \mid degree(p) = 4\ \}$
   (b) $V = \{p(x) \in \mathcal{P}(\mathbb{R}) \mid\ \int_0^1 (p(x))^2\ dx \leq 1\}$
   (c) $W = \{p(x) \in \mathcal{P}(\mathbb{R}) \mid p(0) = p(1)\}$
2. Let $U = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + w = y + z\}$.
   (a) Show that $U$ is a subspace of $\mathbb{R}^4$.
   (b) Show that $\mathbb{R}^4 = U \oplus \mathbb{R}(0, 0, 0, 1)$.
   (c) Find a basis for $U$. (You must justify why it is a basis.)
3. Prove that $\{v_1, \dots, v_n\}$ is linearly independent if and only if $\{v_1, \dots, v_{n-1}\}$ is linearly independent and $v_n \notin span(v_1, \dots, v_{n-1})$.
4. Determine whether or not $\{(x, y, z) \mid 5x^2 - 3y^2 + 6z^2 = 0\}$ is a subspace of $\mathbb{R}^3$.
5. Prove or give a counterexample: If $\{v_1, \dots, v_n\}$ is a linearly dependent set of vectors, then for all $i$, $v_i$ is a linear combination of the other vectors in the set.
6. Recall, the cartesian product of two sets $A$ and $B$ is the set of all ordered pairs $(a, b)$ with $a \in A$ and $b \in B$. That is $A \times B = \{(a, b) \mid a \in A, b \in B\}$. If $U$ and $V$ are $F$-vector spaces, show that $U \times V$ is also an $F$-vector space. If $\dim U = m$ and $\dim V = n$, what is the dimension of $U \times V$?