Show all your work. Partial credit will be given only if work is relevant and correct. *Please make your work as clear and easy to follow as possible.* You might want to put scratch work on the back of every sheet, and put neat clean work on the front of every sheet. You don’t need to simplify your answers but you need to justify them.

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Exercise 1.

Verify Stoke’s theorem for the vector field \( \mathbf{F} = (xy, xz, z + 1) \) where \( S \) is the surface defined by \( z = x^2 + y^2 \) and \( z = 1 \).
Exercise 2.

Find the Fourier series of the function

\[ f(x) = \begin{cases} 
2 & \text{if } 2k\pi \leq x < (2k + 1)\pi \\
0 & \text{if } (2k + 1)\pi \leq x < (2k + 2)\pi 
\end{cases} \]

where \( k = 0, \pm 1, \pm 2, \ldots \).

\[ f(x) = \]
Exercise 3.

Let $S$ be the upper half surface of the sphere $x^2 + y^2 + z^2 = 4$. Compute

$$\iint_S z^2 \, dS = \text{______________________________}$$
Exercise 4.

Expand the function $f(x) = x^2 e^{2x}$ into a power series around the point $x = 0$. 
Exercise 5.

Determine whether the following series converge (Justify your answer):

1) \[ \sum_{k=1}^{\infty} \frac{(-1)^k}{2k + 1} \]

2) \[ \sum_{k=1}^{\infty} \frac{e^k}{e^{2k} - 1} \]

3) \[ \sum_{k=1}^{\infty} \frac{1}{2k^2 - 1} \]
Exercise 6.

Find the solution for the wave equation:

\[
\begin{align*}
  u_{tt} - 25 \, u_{xx} &= 0 \\
  u(0, t) &= 0 \quad t \geq 0 \\
  u(\pi, t) &= 0 \quad t \geq 0 \\
  u(x, 0) &= 2 \sin(x) \quad 0 \leq x \leq \pi \\
  u_t(x, 0) &= 3 \sin(2x) \quad 0 \leq x \leq \pi 
\end{align*}
\]

\[u(x, t) = \]
Exercise 7.
Determine for which $x$ the following series converges:

$$\sum_{k=1}^{\infty} c^k x$$

Compute its value for all the values of $x$, for which it converges.
Exercise 8. (Extra)

Let $S$ be the surface defined by $z = x^2 + y^2$, $z = -x^2 - y^2$ and $x^2 + y^2 = 1$ and let $F = (x + y, y + z, x^2 + z)$.

Compute

$$\int \int_S F_n \, d\sigma = \text{______________}$$