# Math 8 - Solutions to Home Work 2
Due: October 11, 2007

1. **Every/Only.** Sometimes sentences with the words "only" and "every" can be conditional statements in disguise. For example, "Every even number is a multiple of two." can be rephrased as "If a number is even, then it is a multiple of two."

   Rewrite the following propositions as conditional statements in the form "If ..., then ...".

   **Solution.** (Other possibilities exist.)

   (a) "Every small dog is a puppy." = "If a dog is small, then it is a puppy."

   (b) "Everybody who speaks Spanish also speaks Italian." = "If you speak Spanish, then you speak Italian."

   (c) "I eat all vegetables." = "If it is a vegetable, then I will eat it."

   Similarly, for "only", the sentence "I only eat spaghetti." can be rephrased as "If it is not spaghetti, then I will not eat it." or "If I eat something, then it is spaghetti." Notice that the second rephrasing is just the contrapositive of the first.

   Rewrite the following as conditional statements in the form "If ..., then ..." in two equivalent ways (i.e., your two sentences should be the contrapositives of one another).

   **Solution.** (Other possibilities exist.)

   (d) "I only have class on Fridays." = "If I have class today, then it is Friday." = "If it is not Friday, then I don't have class."

   (e) "Only squirrels have fluffy tails." = "If something has a fluffy tail, then it is a squirrel." = "If it is not a squirrel, then it doesn't have a fluffy tail."

   (f) "It only rains in the mornings." = "If it is raining, then it is morning." = "If it is not morning, then it isn't raining."

2. (Optional, but recommended) You may have noticed that the word "only", like the word "unless" in sect. 1.3 ex. 4, can be ambiguous. For example, does "I only eat spaghetti." really mean the same thing as "If I eat something, then it is spaghetti."? Or does it mean "I eat something if and only if it is spaghetti."?

   Think about this question for (d)-(f) above. In each case, which interpretation of "only" seems more appropriate? Also, how does the meaning of "I only eat spaghetti." compare to the meaning of "I will not eat it, *unless* it is spaghetti." ? Can you rewrite the sentences (d)-(f) using "unless"? Finally, try to decide whether the ambiguity with "only" is equivalent to the ambiguity with "unless" that you explored in 1.3 ex. 4.

   **Solution.** (d) is probably best understood as "I have class today if and only if it is Friday." It can also be said "I don't have class today, *unless* it's Friday."

(e) probably means "Something has a fluffy tail if and only if it is a squirrel.", but the other interpretation makes sense here too. It can also be said "It's not a squirrel, *unless* it has a fluffy tail."

(f) is most likely just a conditional, and not a biconditional: "If it's raining, then it's morning." It could also be phrased "It's definitely not raining, *unless* it's morning."

Yes, it does appear that the ambiguity with "only" is similar to that with "unless", since the ambiguity in both types of statements comes from the fact that they can be understood either as conditionals or as biconditionals. It is hard to express this symbolically since "only" has not been used as a logical connective above.

3. Recall that **Modus Ponens** is the rule of inference that says "If $P$ and $P \Rightarrow Q$ are true propositions, then $Q$ is also true." Rewrite this conditional statement symbolically. Then, check that what you get is a tautology by constructing a truth table. (In fact, any logical rule of inference corresponds to a tautology.)

**Solution.** $[P \wedge (P \Rightarrow Q)] \Rightarrow Q$. We see that this is a tautology since its column contains only T's in the truth table below.

| $P$ | $Q$ | $P \Rightarrow Q$ | $P \wedge (P \Rightarrow Q)$ | $[P \wedge (P \Rightarrow Q)] \Rightarrow Q$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |