

## Math 8 - Homework #3

Due: October 18, 2007

For exercises 1-3, do the following:

- (a) Rewrite the given proposition as a conditional (if-then) statement.
- (b) Prove the proposition or give a counterexample.
- (c) If you prove it, say whether your proof is direct, indirect or by contradiction.

1. The sum of any two rational numbers is rational.
2. The product of any two irrational real numbers is irrational.
3. For every odd prime number  $p$ , at least one of the numbers  $p + 2$ ,  $p + 4$  is also prime.
  
4. Let  $n$  be an integer. Prove that if  $n^2$  is even, then  $n$  is even.
5. Prove that the sum of two integers  $a$  and  $b$  is even if and only if  $a$  and  $b$  are both even or both odd.
6. Prove that 5 is a prime number.
  
7. **Extra Credit.** (turn in separately to me) Prove or disprove: If  $x$  and  $y$  are positive irrational real numbers, then  $x^y$  is irrational. (Hint: we know from class that  $\sqrt{2}$  is irrational.)