Name: ______________

Perm No.: ______________

Math 8 - Practice Final Exam

Spring 2009

Instructions:

- This exam consists of 9 problems totalling 90 points, and one bonus problem worth up to 10 points.
- You must show all your work and fully justify your answers in order to recieve full credit. Partial credit will be given for work that is correct and relevant. Your proofs will be graded for clarity and organization, in addition to correctness.
- No books, notes or calculators are allowed.
- Write your answers on the test itself, in the space alotted. You may attach additional pages if necessary.

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Bonus	
Total	

1. Write each of the following sets in set-builder notation $\{x \in U \mid P(x)\}$, where U is a set and $P(x)$ is a proposition depending on x.

   (a) $A = \{0, 4, 16, 36, 64, 100, \ldots\}$

   (b) $B = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, \ldots\}$

2. Are the following propositions True or False? Give brief justifications for your answers. (The domain of interpretation for all variables is $\mathbb{Z}$.)

   (a) $\forall x\ \exists y\ \exists z\ (x + y = 2z)$.

   (b) $\forall x\ \exists y\ (xy = x + y)$.

3. Write the following statements using symbols only, and no words. (You do not have to prove them.)

   (a) "Every rational number is a real number."

   (b) "The product of any two odd integers is odd."

4. For two subsets A, B of a set U, let $A \bigodot B = \widetilde{A \cup B} = U - (A \cup B)$. Draw Venn diagrams illustrating the following subsets of U.

   (a) $A \bigodot B$

   (b) $\tilde{A} \bigodot \tilde{B}$ (note $\tilde{A} = U - A$ is the complement of A.)

   (c) $(B - A) \bigodot (A - B)$.

5. Give examples of functions $f : A \to B$ and $g : B \to C$ with the stated properties, or briefly explain why none can exist. (Be sure to specify the sets A, B, C in your examples.)

   (a) f is one-to-one, but $g \circ f$ is not one-to-one.

   (b) $g \circ f$ is one-to-one, but f is not one-to-one.

6. Consider the relation $\equiv$ on $\mathbb{R}$, defined by

$$a \equiv b \Leftrightarrow a - b \in \mathbb{Z}$$

for any $a, b \in \mathbb{R}$.

(a) Show that $\equiv$ defines an equivalence relation on $\mathbb{R}$.

(b) Describe the equivalence classes of 1 and $\frac{1}{2}$.

7. Let $A = \{0, 1, 2\}$, and let B be the set of all functions $f : A \to A$. Let $e : B \to A$ be the function defined by

$$e(f) = f(0) \quad \text{for any function } f : A \to A.$$

Is the function e one-to-one, onto, or bijective? Justify your answer. (You should explain why it **does** have any of these properties, and ALSO why it **does not** have the other properties)

8. Suppose that A and B are sets with equal power sets, that is, $\mathcal{P}(A) = \mathcal{P}(B)$. Prove that $A = B$.

9. Let x be a real number such that $x \geq 1$. Prove by induction that

$$x^n - 1 \geq (x-1)^n$$

for any integer $n \geq 1$.