Math 8 - Some Games
March 30, 2009

The "games" described below are for two players (although they probably become more interesting if there are 3 or more players). Break up into pairs and play each game several times, alternating who gets to go first. While playing, think about and discuss the following questions:

1. Is there an optimal strategy? If so, what is it?.
2. Can the player who goes first always win? What about the player who goes second?
3. How would you change your strategy if the game is modified slightly? (See below for suggested modifications.) Do these modifications affect who will win?

Stick-Removal Games. In the following games, players take turns removing some number of sticks (or crossing out lines). The player to remove the last stick loses.

A) The game starts with 11 sticks in a row. Players take turns removing 1 stick at a time until all sticks are gone. The player to take the last stick loses. (Modification: Try starting with a different number of sticks.)

B) Again, start with 11 sticks in a row. Players take turns removing either 1, 2 or 3 sticks at a times until all sticks are gone. The player to take the last stick loses. (Modification: Try starting with a different number of sticks.)

C) **Nim.** Start with 3 rows of sticks, containing 1, 3 and 5 sticks. Players take turns removing any number of sticks from one of the rows until all sticks are gone. The player to take the last stick loses. (Modification: Try adding a fourth row with 7 sticks. More generally, you can try varying both the number of rows and the number of sticks in each row in the starting arrangement.)

Dodgeball. The first player has a 6×6 grid of squares, and the second has a single row of 6 squares. The first player makes a "throw" by filling in the first row of the grid with X's and O's. The second player then "dodges" by filling in the first square of his/her row with an X or an O. Play continues in this way until both grids are filled (6 turns each). At the end, the thrower wins if one of the rows in his/her grid matches the dodger's row, while the dodger wins if his/her finished row is different from all the rows in the thrower's grid. (Modification: to make the game a little harder, try counting the *columns* (top to bottom) of the thrower's grid as additional throws. So the thrower wins if, at the end, any of the rows or columns of his/her grid matches the dodger's row.)