Homework I
Financial Mathematics and Economics
Professor: Paul J. Atzberger
Due: Thursday, September 8th
Please turn all homeworks into my mailbox in Amos Eaton Hall by 5:00pm.

**Problem 1:** Use probability theory to model two fair coins which are flipped simultaneously:

a) Give the sample space, collection of events, and probability measure assuming each coin has an equal probability of coming up heads vs tails.

b) What is the probability that at least one coin with heads is observed?

c) What is the probability that only one coin is tails?

**Problem 2:** Two people have a one-on-one basketball match, which ends when one of the players has beaten the other three times. Assume the players are well matched and have an equal chance of winning each game.

a) At most how many games need to be played to determine a winner?

b) What is the probability of the match lasting exactly three games?

c) What is the probability of the match lasting more than three games?

d) Let X be the number of games played in the match. What is the expectation of X? What is the variance of X?

**Problem 3:** A casino has slot machines which accept a variable payment $m$ each play and pay double or nothing. The probability of winning is 49/100.

a) What is the expected payoff if you make 100 bets of $2 each? What is the variance of this payoff?

b) What is the expected payoff if you make only one bet of $200? What is the variance of the payoff?

c) Which one is the best bet for the player (if they are forced to play)? Which is best for the casino? Why?

**Problem 4:** Suppose that a stock initially priced at $92 has prob-
ability $5/8$ to go up by $8$ and the probability $3/8$ to go down $8$ each period $\tau$.

a) What is the expected total return of the stock over a period $\tau$?
b) What is the expected rate of return over the period $\tau$?
c) Let the non-compounding interest rate on a CD savings account be $2.17\%$ over the period $\tau$. Which is the better investment choice neglecting taxes and other transaction costs? State why?

**Problem 5:** A die is thrown five times having face values given by the random variables $X_1, X_2, X_3, X_4, X_5$. Let $Y = X_1X_2X_3X_4X_5$ and $Z = X_1 + X_2 + X_3 + X_4 + X_5$. Hint: make use of independence and properties of expectations to avoid tedious calculations.

a) Compute the expected value of the random variables $Y$ and $Z$.
b) Compute the variance of the random variable $Y$ and $Z$.
c) What is the covariance and correlation of $Y$ with $Z$.

**Problem 6:** A wine tasting and classical music concert is planned at an outdoor park. The organizers plan to spend $1$ million on the event. The tickets and sponsors brought in sales of $4$ million, but will have to be refunded if there is rain. A local company offers insurance costing $0.50$ for a policy that pays $1.00$ in the event of rain. Assume that the probability of rain is $50\%$.

a) Compute the expected payoff and variance when the organizers buy $x$ units of rain insurance.
b) How much insurance should the organizers buy? Why?

**Problem 7:** Write a matlab code which computes the histogram of the random variable $Y = \sqrt{X}$ where $X$ is the uniform random variable on the interval $[0, 1]$. Hint: You may find the following commands useful: “rand”, “hist”, “sqrt”. To get help on a command type “help command_name”.

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