Homework 6
Introduction to Numerical Analysis
http://www.math.ucsb.edu/~atzberg/fall2006/index.html
Professor: Paul J. Atzberger
Due: Wednesday, November 22nd
All homeworks are to be turned in at the beginning of class Tuesday or in my mailbox (South Hall 6th Floor) by 5pm Wednesday before the holiday break.

Problem 1: (problems in the book by Faires and Burden)
3.1: 1, 6a,c,d, 10a,c,d, 11, 25, 27.

Problem 2: (Marginal Product of Labor) Let us consider the case of a company which bakes loaves of bread for the restaurants and supermarkets in a city. In this exercise you will determine the optimal number of employees to hire in order to maximize the profit of the company. Let us assume the following model for how the firm operates.

(i) Given the ovens and space available in the bakery, for $n$ employees the number of loaves of bread produced per day is given by $g(n) = \frac{500n}{n + 20}$.

(ii) Each loaf of bread sells for $p = $10.00.

(iii) Each employee costs $c = $30.00 per day.

a) The profit $P(n)$ when the company has $n$ employees is given by the revenue $R(n)$ (total income) minus the operating costs $C(n)$. This can be expressed as:

$$P(n) = R(n) - C(n).$$

Under our assumptions for how the bakery operates we have $R(n) = p \cdot g(n)$ and $C(n) = c \cdot n$ which gives a profit:

$$P(n) = p \cdot g(n) - c \cdot n.$$  
(2)

For this model, make a plot of $P(n)$ vs $n$.

b) A basic concept from macroeconomics used to explain the size of a labor force is the Marginal Product of Labor. This notion captures the trade-offs a manager at a profit maximizing company must deal with between the benefit of having a certain number of employees working at the firm and the costs associated with having each employee. The Marginal Product of Labor postulates that the size of the labor force at a firm will continue to grow while this increases the firm’s profit. In other words, the firm will continue to grow until adding employees has an adverse effect on profits. Mathematically, the optimal size of the firm can be characterized as the largest $N$ for which $P(N) - P(N - 1) > 0$, while $P(N + 1) - P(N) < 0$.

Use the Bisection method, implemented in the previous assignment, to compute the optimal size $N$ of the baking company’s labor force.
(Hint: compute a zero $n^*$ of $P(n) - P(n - 1) = 0$ and check the condition above for nearby integer values to determine $N$).

c) How does changing the production function to $g(n) = 500n^2/(n^2 + 800)$ change the optimal size? Specifically, what now is the optimal $N$?

d) What happens at the firm if bread becomes more in demand and the price of a loaf increases to $p = $15.00? What about if demand drops and the prices decreases to $p = $5.00? In which case do you expect wage cuts or layoffs? (use $g$ from (i) and give $N$ in each of the cases)

e) Please print out only your Matlab/Octave source code (script file) and attached it at the end of your assignment. No need to print the execution output.