Problem Set #2: Solutions

Page 44, Section 1.3

3. (a) TRUE (b) FALSE (Its actually Lansing) (c) FALSE (Clearly, Massachussetts is not the capital of Boston) (d) FALSE (Its actually Albany... if you didn’t know that already...)

**10. (a) ∃x(C(x) ∧ D(x) ∧ F(x)) (b) ∀x(C(x) ∨ D(x) ∨ F(x)) (c) ∃x(C(x) ∧ ¬D(x) ∧ F(x)) (d) ¬∃x(C(x) ∧ D(x) ∧ F(x)) (e) (∃x(C(x)) ∧ (∃x D(x)) ∧ (∃xF(x))). Note here that ∃x(C(x) ∨ D(x) ∨ F(x)) is not correct because according to the question, at least one student has a cat, a dog or a ferret, while according to the last (incorrect statement), even if all the students in the class had cats and no dogs or ferrets, the statement would still be true.

**57.

a ∀x(P(x) → ¬Q(x))

b ∀x(R(x) → ¬S(x))

c ∀x(¬Q(x) → S(x))

d ∀x(P(x) → ¬R(x))

e (d) does indeed follow from (a), (b) and (c). To see that consider (a) which suggests that all babies are illogical. Then by (c), babies are despised, since all who are illogical are despised. However, by (b), those who can manage crocodiles are not despised. Since babies are despised, they cannot manage crocodiles. This lesson is, however, lost on the Crocodile Hunter.

Page 51, Section 1.4

1. (a) For every real number x, there exists another real number y such that x is less than y. (b) For all real numbers x and y, if x and y are greater than or equal to zero, their product is greater than or equal to zero. (c) For all real numbers x and y, there exists a real number z such that z is equal to the product of x and y.

**17.

a ∀u∃b (A(u, b) ∧ ∀m(b ≠ m → ¬A(u, m))), i.e., for every user u, there exists a mailbox b, such that u has access to b and for every other mailbox m that is not the same as b, u does not have access to m. This can also be written using the uniqueness quantifier as ∀u∃!bA(u, b).

b ∃p∀e R(p, e) → R(kernel, e) i.e., there is a process running for all error conditions if the kernel is running when the error condition is in effect.

c ∀u∀w E(w, .edu) → A(u, w)

d ∃x∃y ((x ≠ y) ∧ (x < 0 ∨ (y ≥ 0) ∧ (z = x) ∨ (z = y)))). The bi-implies is required because two systems monitor every server and every server is monitored by exactly two server. Note that the uniqueness quantifier cannot be used, because the two servers many not be unique, ie the same two servers may also monitor some other system.

**20.

a ∀x∀y (x < 0 ∧ y < 0) → xy < 0

b ∀x∀y (x > 0 ∧ y > 0) → x + y > 0

c ¬∀x∀y (x < 0 ∧ y < 0) → x − y < 0

d ∀x∀y |x + y| ≤ |x| + |y|