Math 34A
Week 2
13 October 2017

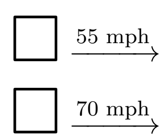
Math 34A — Week 2

28.

(a) A highway patrolman traveling at the speed limit is passed by a car going 15 mph faster than the speed limit. After one minute, the patrolman speeds up to 100 mph. How long after speeding up until the patrolman catches up with the speeding car. The speed limit is 55 mph.

(b) Same question, but this time the patrolman speeds up to a speed of v mph ($v > 70$).

(a) This problem seems complicated. There is a lot going on, so it may be easier to break it up into parts. If we read the problem again, there is a sort of natural halfway point in the problem. The problem begins with a speeding car going faster than the patrolperson, and then it switches to the patrolperson going faster than the speeding car. In the second situation, we want to find how long it takes for the patrolperson to catch up to the speeding car. This sounds like $d = rt$, but we don't know what d is, so we don't know how to find the time. However, the first part of the problem gives us some information that helps us find the distance between the cars. Let us begin with the initial situation then. The problem begins with both cars in the same position, as depicted below.



Let's call this initial time $t = 0$. At this time, we see the lower car zooming by the other car, and this situation continues for 1 minute. So, at $t = 1$ minute, the two cars are some distance apart from each other. How far? Well, we can use $d = rt$. We have $t = 1$ minute. We need a rate, r. It may be tempting to say that the rate is 70 mph because that's how fast the car that is pulling away is going, but that's not quite right because the other car is also moving. The rate we want is *how fast the distance between them is increasing*. How fast is this? Well, it's just the difference in their speeds, $r = 70 - 55 = 15$ mph. Thus, after one minute, we have a distance of

$$d = 15 \text{ mph} \cdot 1 \text{ minute}$$

between the two cars. Remember to convert units when doing this computation!

So, we now have a distance, but I don't even remember why I started doing any of this. Oh right! If we look back, we remember that what we want is how long it takes for the patrolperson to catch up. After 1 minute (and so after the situation we considered above), we know the distance between the cars is whatever we calculated above (I will call it d), and we know that the patrolperson speeds up to 100 mph. The situation now looks like this:

where the distance between the two cars is d. To find how long it takes to catch up, we can again use our formula $d = rt$. To find a time, we need to rearrange to get

$$t = \frac{d}{r}$$

Now, we know d because we did that already, but we need r. Again, this r is the rate at which the distance between the two cars is changing. Can we find this rate and then plug in d and r to find how long it takes for the patrolperson to catch up to the speeding driver? Remember to match units!

(b) We can do the same thing as in part (a), but this time, we have v instead of 100.

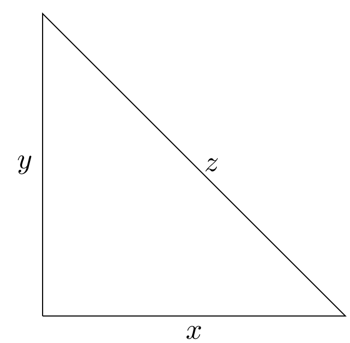
$\square$

25.

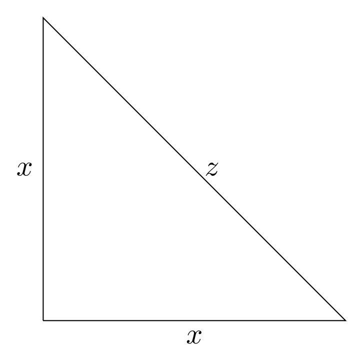
(a) A right-angled triangle has a 45° angle. If the area is 2 cm^2, what is the length of the perimeter?

(b) Same question, but this time the area is A cm^2.

(a) We start by drawing a picture:



Since this is a 45-45-90 triangle (or alternatively, since it is isosceles), we know that sides x and y are actually the same length. We can redraw the picture as



Now, we want to find the perimeter of the triangle, $P = x + x + z$, but all we are given is an area, so somehow, we need to convert. Well, we know the area of a triangle as a function of base and height to be

$$A(b,h) = \frac{1}{2}bh,$$

which we can use to relate A to x! In this case, we see that $b = h = x$, so

$$\begin{aligned} A &= \frac{1}{2}x \cdot x \\ &= \frac{1}{2}x^2. \end{aligned}$$

Now, we know the area of the triangle to be 2, so we can set

$$2 = \frac{1}{2}x^2$$

and solve for x! This gives us a side length with which to find the perimeter. (Hint: the Pythagorean theorem can help us find z.)

(b) We do the same thing, but we don't substitute 2 for A. When we solve for the side length, it will be in terms of A, but that's fine! Just do the same thing, and you will end up with a perimeter that depends on the area. This equation is now the perimeter of *any* 45-45-90 triangle with a known area, not just one with area 2!

$\square$

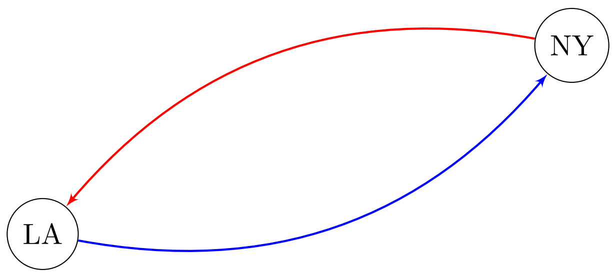
62. The radius of the Earth is about 4,000 miles. One rope is laid on the ground all the way around the equator. A second rope is placed exactly 5 feet above the second rope all the way around the equator. How much longer is the first rope in feet?

Note that this problem is NOT asking us to consider a rope 5 feet *north* of the first rope. The second rope is hovering 5 feet above the earth all the way around. So, if the radius of the earth is r, this new ring has radius $r + 5$. Draw a picture to see this for yourself.

$\square$

47. An airplane departs from LA and flies to NY every 30 minutes. The trip takes 5 hours and 5 minutes. An airplane takes off from NY at the same time that one takes of from LA and flies to LA at the same speed. How many planes does it pass going in the opposite direction?

As always, pictures are our friends. Let's draw the situation:



Let's call the plane that goes from NY to LA plane p and the plane that takes off from LA at the same time plane q. When plane p first departs on the red route, there are already planes in the air on the blue route. The journey is just over 5 hours long. Since planes leave every half hour, there are ten planes that left before plane q along the blue route that are still in the air. Plane p must pass all of these planes, so there are at least 10. At the halfway point of the journey, plane p passes plane q, so that is one more crossing. Finally, by the time plane p arrives in LA, plane q has arrived in NY, and there are again 10 planes behind it (think about why this is true!). Can we now find the total number of planes that were passed?

□

19. There are more than 1/4 million species of beetle. Assume the average length of a beetle is 1 cm and the average walking speed is 10 cm per second. These beetles walk up a gang-plank that is 5 meters long onto an ark. They do this in pairs, side by side, one male and one female from each species. The pairs of beetles are spaced 2 cm apart. How many hours will it take for 1/4 million species of beetles to embark onto the ark once the first pair starts up the gangplank?

We have 250,000 beetles that need to make it onto the ark. We could think about this problem by considering how many beetles can walk up the plank at a time. We could also just imagine them as a line 250,000 beetles long Unfortunately, this unit is not the most helpful, so lets convert beetles to centimeters. Each beetle is 1 cm long and they are spaced 2 cm apart. So, each beetle takes up three centimeters. We can use this information to find the total length of the line of beetles. This is sort of a "distance". Of course, we also have a rate: 10 cm/second. Can we now find how long it takes for them all to get on the ark? Remember to convert units!

□

51. An airline sells all the tickets for a certain route at the same price. If it charges 200 dollars per ticket it sells 10,000 tickets. For every 15 dollars the ticket price is reduced, an extra thousand tickets are sold. Thus if the tickets are sold for 185 dollars each then 11,000 tickets sell. It costs the airline 100 dollars to fly a person.

(a) Express the total profit P in terms of the number n of tickets sold.

(b) Express the total profit P in terms of the price p of one ticket.

Both parts of this problem are asking us for an expression for total profit, P, so let's just focus on that for now. We need to find out what the total profit of this airline is. Well, in general, profit is just how much you earn minus whatever costs were involved. In this case, if n is the number of tickets sold and p is the price per ticket, we can simply write the total earned as

$$\text{Total earned} = np.$$

Of course, we need to now subtract our costs. The problem tells us that it costs \$100 to fly a person. If we fly n people, our costs are

$$\text{Total cost} = 100n.$$

So, our total profit is just

$$P = np - 100n.$$

Alright. We've found what the problem wanted us to find, namely the total profit. But we don't want profit in terms of both n and p. This is not as helpful to us since we can't easily find our profits in terms of one thing we have control over. So, we want to rewrite this so that (a) it's just in terms of the number of tickets sold and (b) it's just in terms of the price per ticket. To do this, it would be helpful if we had an equation relating n and p. Well, luckily, half the problem statemnent is devoted to telling us what this relationship is!

We could reason about the relationship between n and p, and this will give us the correct answer, but a more systematic approach that may be helpful when we're stuck is to recall the equation of a line: $y - y_0 = m(x - x_0)$. This equation graphs a line. It is appropriate here because the relationship between the price of a ticket and the number of tickets sold is a line: for every 15 dollar the price changes, the number of tickets changes by a fixed amount (namely 1000). In order to use this equation, we need a point (x_0, y_0) and a slope m. Well, we are told that the point $(200, 10000)$ is on the line, so maybe we set

$$x_0 = 200, y_0 = 10,000.$$

To find a slope, we see that the problem tells us that for each $-\$15$ change in price, there is a $+1,000$ change in number of tickets. This gives us a slope of

$$m = \frac{\text{rise}}{\text{run}} = -\frac{1000}{15}.$$

Can we now find an equation relating n and p? Once we have it, we can just solve and plug into our profit equation to eliminate a variable.

□