Math 34A — Week 9

Derivative Practice Find the derivative of the following expressions with respect to x.

(a) $(x^2+1)^2$

(b) ax^2+bx+c

(c) $\frac{x^2+2x+1}{x+1}$

(d) $\sqrt{x}$

(e) $\left(\frac{1}{\sqrt[3]{x}}\right)^2$

(f) $\frac{x^2+4x+3}{\sqrt{x}}$

(g) $\left(\frac{x^2+4x+3}{\sqrt{x}}\right)'$

(h) $(5x)^4$

(i) $\frac{(5x)^4}{4!}$

(j) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

(k) e^x

(l) e^{-x}

(m) e^{cx}

(n) 10^x

(o) 5^{ex}

4.3 Find the equation of the line through $(2, a)$ and $(5, b)$.

4.11 Find where the line which passes through the two points $(1, 2)$ and $(3, 5)$ intersects the line through $(2, 1)$ and $(5, -6)$.

4.22 If y is proportional to x and x is 4 when y is 42, then what is y when

(a) $x = 8$?

(b) $x = 2$?

(c) $x = a$?

(d) $x = a^2$?

(e) $x = 0$?

(f) $x = (a + b)$?

(g) $x = \frac{1}{w}$?

6.7 Consider the function $f(x) = \sqrt{x}$.

(a) Use the tangent line approximation at $x = 4$ to approximate $f(4.4)$.

(b) What is the percentage error in your answer to part (a)?

8.19 A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$9 per m^2. Material for the sides costs \$9.6 per m^2. Find the dimensions of the container which will minimize cost and the minimum cost (ie. find the base length, base width, height, and the resultant minimum cost).

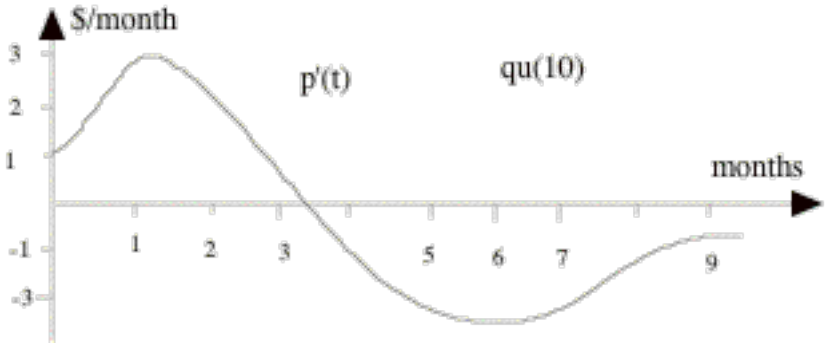
8.20 Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r.

8.22 Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola $y = 20x^2$.

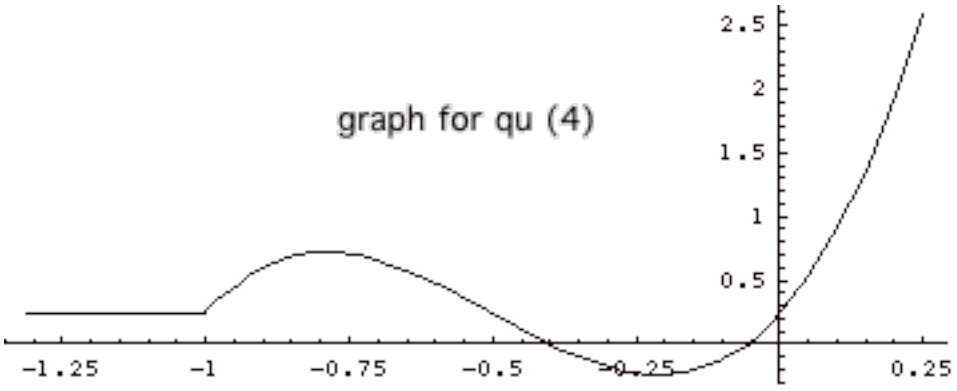
7.57 Coca-Cola has hired you to design a new can for their soda. They will make the top of the can separately, so you are in charge of designing a cylindrical metal can with no lid. It is to have a volume of 64π in^3. What height and radius should you advise in order to minimize the amount of metal used? What if we wanted to maximise the amount of metal used?

6.40 The number of items sold at a price of x dollars per item is $2000 - 300x^3$. It costs 6 dollars to make the item. What price should be charged to make the most profit?

7.24 The price of IBM stock t months after you buy is $p(t)$ dollars. Below is the graph of $p'(t)$ the derivative of $p(t)$. After how long should you sell for the most profit?



7.31 Refer to the graph below for this problem. On what intervals does the graph have positive second derivative? At what points is the derivative 0?



8.13 Plane A flies at a constant speed from New York to Los Angeles along a route which is 2000 miles. Plane B flies in the opposite direction at a constant speed which is 100 mph faster than plane A. Plane B takes off one hour after plane A. They land at the same moment. How far are they from Los Angeles when they pass?

8.24 A boat leaves a dock at 2:00 P.M. and travels due south at a speed of 20 km/h. Another boat has been heading due east at 15 km/h and reaches the same dock at 3:00 P.M. How many minutes past 2:00 P.M. were the boats closest together?

4.54 The half-life of carbon-14 is 5730 years. A bone is discovered which has 2 percent of the carbon-14 found in the bones of other living animals. How old is the bone?

4.56 On the planet Maximillian live Sprogs and Graks. Initially there were 3200 sprogs and 400 Graks. The population of Sprogs doubles every 10 years and that of Graks doubles every 5 years.

(a) How many Graks were there after 2.5 years?

(b) When are there as many Sprogs as Graks?

5.37 A tank initially contains 1000 liters of pure water. Then water containing 5 mg of detergent per liter starts to enter the tank at the rate of 30 liters per hour.

(a) How long until the average concentration of detergent in the tank is 2 mg per liter?

(b) How long until the average concentration of detergent in the tank is x mg per liter?

(c) Sketch a graph showing the function you obtained in (b). Put x on the horizontal axis and t on the vertical axis.

(d) What does your answer to part (b) give when $x = 7$. Do you notice anything strange? Can you explain this?

7.22 Air is pumped into a spherical balloon, so the balloon expands. The volume of a sphere of radius R is $\frac{4\pi R^3}{3}$. If the radius of the sphere after t seconds is 2t centimeters, at what rate is air being pumped in when t=5? (Hint: the rate air is pumped in equals the rate that the volume of the sphere increases).

9.4 Solve the following equation for w in terms of the other quantities.

$$\frac{6}{w-1} + \frac{a}{b+a} = -6$$

9.36 A baseball team plays in a stadium that holds 64000 spectators. With the ticket price at \$11 the average attendance has been 25000. When the price dropped to \$10, the average attendance rose to 32000.

(a) Find the demand function $p(x)$, where x is the number of the spectators. (Assume $p(x)$ is linear.)

(b) How should ticket prices be set to maximize revenue?