

1. Amtrak Tickets

Amtrak sells all the tickets for a certain route at the same price. If they charge \$118 per ticket, then they sell 2,000 tickets each week. For every \$15 the ticket price is reduced, an extra five hundred tickets are sold each week. It costs Amtrak \$40 per person to run the train. What price will generate the greatest weekly profit for Amtrak?

2. Half-Life

The half-life of carbon-14 is 5730 years. A bone is discovered which has 22% of the carbon-14 found in the bones of living animals. How old is the bone?

3. Limits

Evaluate the following limits:

(a) $\lim_{t \rightarrow \infty} \frac{4+7t}{2-t}$

(b) $\lim_{x \rightarrow \infty} \frac{11+8x}{x^3+6x}$

(c) $\lim_{x \rightarrow 3} \frac{x^2-9}{x-3}$

4. Sums

Find the following sums:

(a)

$$\sum_{n=2}^7 n^2$$

(b)

$$\sum_{k=1}^{109} 2$$

(c)

$$\sum_{k=1}^2 \left(\sum_{n=1}^3 k \cdot n \right)$$

(d) Write the average of the squares of the first 48 positive integers using summation notation.

5. Planes

Plane A flies at a constant speed from New York to Los Angeles along a route which is 2000 miles. Plane B flies in the opposite direction at a constant speed which is 101 mph faster than plane A . Plane B takes off one hour after plane A . They land at the same moment. How far are they from Los Angeles when they pass?

6. Equations

- (a) Solve for x : $10^{x-1} = 10a^3$.
- (b) On the planet Maximillian live Sprogs and Graks. Initially there were 800 Sprogs and 100 Graks. Sprogs double every 10 years and Graks double every 5 years. How many Graks were there after 2.5 years? When are there as many Sprogs as Graks?

7. Exponential

The temperature in degrees Fahrenheit of a corpse t hours after death is $61 + 37e^{-t/24}$.

- (a) How quickly is the temperature decreasing after 2 hours?
- (b) What is the temperature of the surroundings of the corpse?
- (c) What was the temperature at the point of death?

8. Cars

You and your friend drive from UCSB to San Francisco. Your friend leaves at noon and drives at a constant speed. You leave at 1:15 and drive at a constant speed (not the same speed as your friend). At 2:00 you and your friend are 30 miles apart. At 3:00 you catch up to your friend. How fast was your friend driving?

9. Related Rates

A street light is at the top of a 20 ft tall pole. A woman 6 ft tall walks away from the pole with a speed of 8 ft/sec along a straight path. How fast is the tip of her shadow moving along the ground when she is 45 ft from the base of the pole?

How fast is the length of her shadow increasing?

10. Profit

The number of items sold at a price of x dollars per item is $2000 - 300x^3$. It costs 6 dollars to make the item. What price should be charged to make the most profit?

11. Milk

You want to combine milk that is 1% fat and milk that is 4% fat to get 1 gallon of milk that is 3.5% fat. What volume of milk that is 1% fat should you use?

12. Tangent Lines and Approximation

Consider the function $f(x) = \sqrt{x}$.

1. Use the tangent line approximation at $x = 4$ to approximate $f(4.4)$.
2. What is the percentage error in your answer to part (a)?

13. Writing Equations

Assume a circle of radius r has the same area as a square with side length s . Express the radius of the circle in terms of the length of a side of the square.

14. Proportionality

The time it takes to build a skyscraper is proportional to its height and inversely proportional to the number of construction workers. If it takes 10 workers 2.5 years to build a 20 story building, how long will it take 30 workers to build a 80 story building?

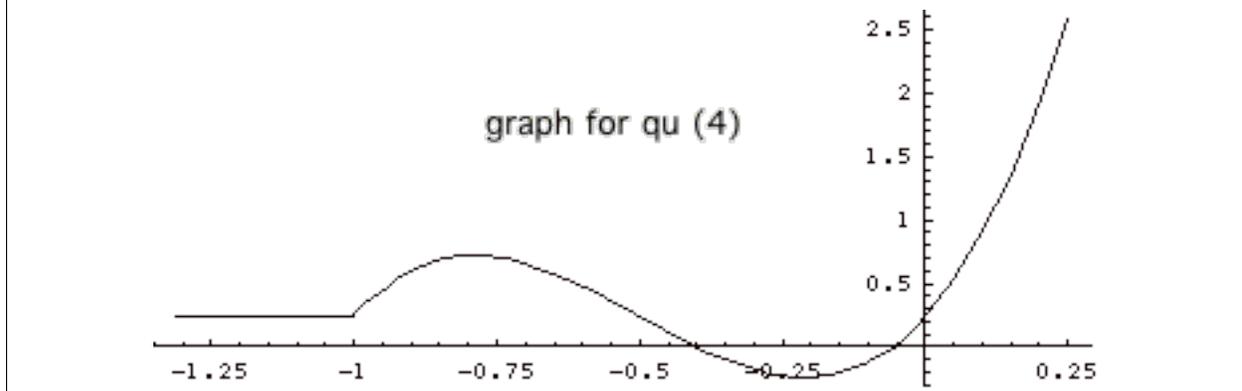
15. Motion

An object is dropped. At t seconds after it is dropped, it is $20 - 5t^2$ meters high.

- (a) How high was it dropped from?
- (b) What was its instantaneous speed 1 second after it was dropped?
- (c) How many seconds did it take to hit the ground?
- (d) What was its acceleration?

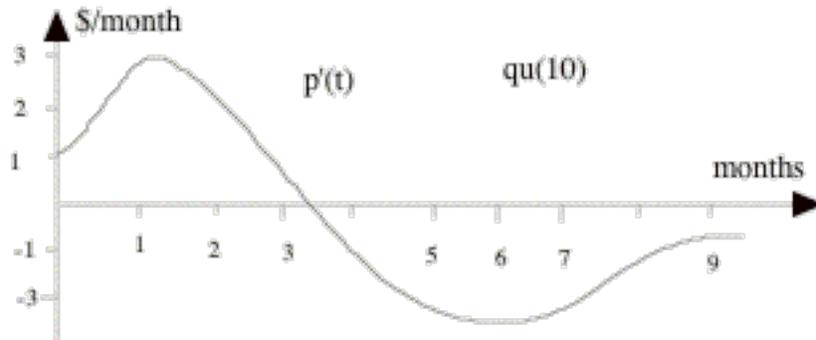
16. Derivatives from Graphs

Refer to the graph below for this problem. On what intervals does the graph have positive second derivative? At what points is the derivative 0?



17. Graphs of Derivatives

The price of IBM stock t months after you buy is $p(t)$ dollars. Below is the graph of $p'(t)$ the derivative of $p(t)$. After how long should you sell for the most profit?



18. Cans

Coca-Cola has hired you to design a new can for their soda. They will make the top of the can separately, so you are in charge of designing a cylindrical metal can with no lid. It is to have a volume of 64π in³. What height and radius should you advise in order to minimize the amount of metal used? What if we wanted to maximize the amount of metal used?

19. Increasing, Decreasing, and Concavity

Let $f(x) = 1 + 3x - x^2$.

1. For what values of x is $f(x)$ increasing?
2. For what value of x is $f(x)$ maximized?
3. Is the graph of $y = f(x)$ concave up or down?

20. Poster

A poster is to have total area of 486 cm^2 . There is a margin of 6 cm at the top and 4 cm at the sides and bottom where nothing can be printed. What width should the poster be in order to have the largest printable area?

Motion and Acceleration

The position of an object on the x -axis after t seconds is given by $x(t) = t^3 - 5t^2 - 7t + 2$ cm to the right of the origin. At time $t = 1$ is the object moving left or right? Is the acceleration increasing or decreasing at that time?

Water Tank

A cylindrical water tank has base a circle of radius 1.7 meters and vertical sides. If water leaves the tank at the rate of 6 liters per minute, how fast is the water level falling in centimeters per hour? (One liter is 1000 cubic centimeters.)