Math 6A

Quiz 1

Name:

Section Time:

Complete the following problems, making sure to SHOW ALL WORK.

- 1. Consider the points A = (-2, 5, 2), B = (1, 1, 1), and C = (7, 4, -3).
 - (a) Find a vector equation of the line passing through points A and B.

To find the equation of this line, we will need a point on the line and a vector along it. Let us just use B as the point on the line and the vector $\overrightarrow{BA} = \langle -2 - 1, 5 - 1, 2 - 1 \rangle = \langle -3, 4, 1 \rangle$ as the desired vector. This gives us a vector equation for the line of $\langle 1, 1, 1 \rangle + t \langle -3, 4, 1 \rangle$ with $t \in \mathbb{R}$.

(b) What is the measure of the angle $\angle ABC$ formed by the line passing through the points A and B and the line passing through the points B and C?

We can easily find angles by using the dot product! We already have \overrightarrow{BA} , so all we need is to find $\overrightarrow{BC} = \langle 7-1, 4-1, -3-1 \rangle = \langle 6, 3, -4 \rangle$. Now, we compute the dot product of these vectors as

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = (-3)(6) + (4)(3) + (1)(-4) = -15.$$

Since $\overrightarrow{BA} \cdot \overrightarrow{BC} = \|\overrightarrow{BA}\| \|\overrightarrow{BC}\| \cos \theta$, we can now find

$$\theta = \arccos\left(\frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{\left\|\overrightarrow{BA}\right\| \left\|\overrightarrow{BC}\right\|}\right) = \arccos\left(\frac{-15}{\sqrt{(-3)^2 + 4^2 + 1^2}\sqrt{6^2 + 3^2 + (-4)^2}}\right)$$
$$= \arccos\left(\frac{-15}{\sqrt{26}\sqrt{61}}\right).$$

(c) Compute the cross product $\overrightarrow{BA} \times \overrightarrow{BC}$.

We've already found the vectors required, so we just compute

$$\overrightarrow{BA} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & 1 \\ 6 & 3 & -4 \end{vmatrix} = (4(-4) - 1(3))\hat{i} - (-3(-4) - 1(6))\hat{j} + (-3(3) - 4(6))\hat{k} = \langle -19, 6, -33 \rangle$$

2. What is the equation of the plane orthogonal to the vector $\vec{n} = \langle 1, 2, 3 \rangle$ and passing through the point A = (3, 2, 1)?

This plane can be described by the set of all vectors that begin at the point A and are orthogonal to the vector \vec{n} . To say this another way, we are looking for all points B such that $\vec{n} \cdot \vec{AB} = 0$. Thus, the plane can be described as all points (x, y, z) such that

$$\langle 1, 2, 3 \rangle \cdot \langle x - 3, y - 2, z - 1 \rangle = 0.$$

Taking the dot product gives us

$$(x-3) + 2(y-2) + 3(z-1) = 0$$

or

$$x + 2y + 3z = 10.$$

Both of these answers are acceptable.