Math 6A Quiz 2

Name:________________________________ Section Time:____________

Complete the following problems, making sure to SHOW ALL WORK. If you're stuck on something, CLEARLY EXPLAINING what you do know will get you partial credit!

1. Consider the points $A = (-2, 5, 2)$, $B = (1, 1, 1)$, and $C = (7, 4, -3)$.

   (a) Find an equation for the plane in $\mathbb{R}^3$ containing all three of these points.

   To find the equation of the plane, we need to first find a vector that is normal to the plane. We can do this by taking the cross product of two vectors in the plane. Since the point B looks easiest to work with, let's use the vectors $\overrightarrow{BA}$ and $\overrightarrow{BC}$. We find these vectors to be

$$\overrightarrow{BA} = \langle -2, 5, 2 \rangle - \langle 1, 1, 1 \rangle = \langle -3, 4, 1 \rangle ,$$
$$\overrightarrow{BC} = \langle 7, 4, -3 \rangle - \langle 1, 1, 1 \rangle = \langle 6, 3, -4 \rangle .$$

   Now we find a normal vector

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 4 & 1 \\ 6 & 3 & -4 \end{vmatrix} = \langle -19, -6, -33 \rangle .$$

   Finally, the plane we wish to find contains B (or any of the points) and is normal to $\vec{n}$:

$$-19(x-1) - 6(y-1) - 33(z-1) = 0.$$

$\square$

   (b) Consider the point $p = (1, 2, 3)$. What is the distance between the plane found in part (a) and the point p? If you did not find the plane in part (a), you may do this problem using the plane $ax + by + cz = C$.

   We already have a normal vector to the plane: $\vec{n} = \langle -19, -6, -33 \rangle$. Let us now find a vector from the plane to the point we care about so we can find the scalar projection onto the normal vector. Since the point B is nice, we'll consider the vector

$$\overrightarrow{Bp} = \langle 1, 2, 3 \rangle - \langle 1, 1, 1 \rangle = \langle 0, 1, 2 \rangle .$$

   Now, we find

$$\left\| \mathrm{Proj}_{\vec{n}} \left(\overrightarrow{Bp} \right) \right\| = \frac{\overrightarrow{Bp} \cdot \overrightarrow{n}}{\| \overrightarrow{n} \|} = \left| \frac{-6 - 33(2)}{\sqrt{(-19)^2 + (-6)^2 + (-33)^2}} \right| .$$

$\square$

2. Find the following limits or show that they don't exist.

(a) $\lim_{t\to 0}\left\langle 17, 1+\sqrt{t}\right\rangle$

(b) $\lim_{t\to 2}\left\langle t^2+t, \frac{1}{t-2}, 1\right\rangle$

(c) $\lim_{(x,y)\to(1,1)} \frac{xy}{x^2+y^2}$

(d) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

(a) Nothing weird going on here. We simply plug in and find

$$\lim_{t\to 0}\left\langle 17, 1+\sqrt{t}\right\rangle = \langle 17, 1\rangle .$$

(b) With this limit, we notice that there is an issue with the second entry of the vector. Notice

$$\lim_{t\to 2^-} \frac{1}{t-2} = -\infty, \qquad \lim_{t\to 2^+} \frac{1}{t-2} = \infty,$$

so the limit does not exist in the second component. Thus, the limit of the entire vector does not exist.

(c) Once again, no problems. We just plug in and find

$$\lim_{(x,y)\to(1,1)} \frac{xy}{x^2+y^2} = \frac{1}{2}.$$

(d) Here, plugging in gives us 0/0. Hmm....suspicious. Let's consider the path $y = 0$. This turns the limit into a single variable limit:

$$\lim_{(x,0)\to(0,0)} \frac{0}{x^2+0} = \lim_{(x,0)\to(0,0)} 0 = 0.$$

On the other hand, consider the path $y = x$. This gives us a limit

$$\lim_{(x,x)\to(0,0)} \frac{x^2}{x^2+x^2} = \lim_{(x,x)\to(0,0)} \frac{1}{2} = \frac{1}{2}.$$

Since the limit is not the same from every path (similar to the situation in part (b)), we conclude that the limit does not exist.

□