Math 6A **Quiz 3**

Name:______________________________ Section Time:__________

Complete the following problems, making sure to SHOW ALL WORK. If you're stuck on something, CLEARLY EXPLAINING what you do know will get you partial credit!

1. Compute the derivatives Df and Dg for the following functions. Note that both of these derivatives are Jacobians, but only one is a gradient.

   (a) $f(x,y,z) = xe^{xy} + \sin(y)$

   (b) $g(x,y) = \left\langle xy + 2y, 2xy^2, \frac{x^2}{y^2} \right\rangle$

   (a) This is a real valued function, so we compute a gradient

$$\begin{aligned} Df(x,y,z) &= \nabla f(x,y,z) \\ &= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle \\ &= \langle e^{xy} + xye^{xy}, x^2e^{xy} + \cos(y), 0 \rangle. \end{aligned}$$

   (b) This function is vector valued, so we compute its Jacobian:

$$\begin{aligned} Dg(x,y) &= \begin{bmatrix} \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} \\ \frac{\partial g_3}{\partial x} & \frac{\partial g_3}{\partial y} \end{bmatrix} \\ &= \begin{bmatrix} y & x+2 \\ 2y^2 & 4xy \\ \frac{2x}{y^2} & \frac{-2x^2}{y^3} \end{bmatrix}. \end{aligned}$$

2. Find the derivative $D_{\vec{u}}f(x,y,z)$ of the function f in the previous problem in the direction of the vector $\vec{u} = \langle -1, 0, 3\rangle$.

As always, we begin computing the directional derivative by finding a unit vector in the direction of $\vec{u}$. To do this, we first need the length of $\vec{u}$, which we find to be

$$\|\vec{u}\| = \sqrt{(-1)^2 + 0^2 + 3^2} = \sqrt{10}.$$

So, our unit vector is

$$\vec{u}' = \frac{\vec{u}}{\sqrt{10}} = \left\langle -\frac{1}{\sqrt{10}}, 0, \frac{3}{\sqrt{10}} \right\rangle.$$

Now, we compute the directional derivative as

$$\begin{aligned}
D_{\vec{u}}f(x,y,z) &= \nabla f \cdot \vec{u}' \\
&= \left\langle e^{xy} + xye^{xy}, x^2e^{xy} + \cos(y), 0 \right\rangle \cdot \left\langle -\frac{1}{\sqrt{10}}, 0, \frac{3}{\sqrt{10}} \right\rangle \\
&= -\frac{e^{xy} + xye^{xy}}{\sqrt{10}}.
\end{aligned}$$