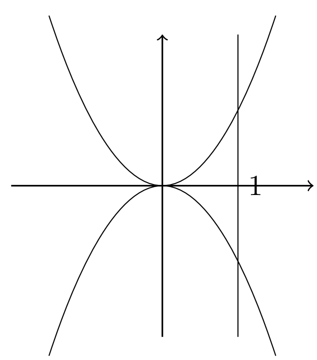
Math 6A **Quiz 5**

Name:________________________________ Section Time:____________

Complete the following problems, making sure to SHOW ALL WORK. If you're stuck on something, CLEARLY EXPLAINING what you do know will get you partial credit!

1. Consider the region D in the x, y-plane bounded by the functions $y = x^2$, $y = -x^2$, and $x = 1$. Consider also the function $f(x, y) = 3 - 2x^2 + y$.
   (a) Sketch the region D in the x, y-plane. Make sure to label axes and significant points of intersection.
   (b) Set up an integral that gives the volume of the solid bound by the region D and the function f.
   (c) Find the volume of the solid by evaluating the integral you found in part (b). If you did not find an integral in part (b), demonstrate you know how to compute such a volume integral if given one. This could involve words, an example with an unrelated integral, or any other method that calls to you.
   (d) Write, draw, or otherwise create something. Potential points for creativity and how much you help me enjoy grading!

(a)



(b) By dividing the region into segments along the x-axis, we get the integral

$$\int_0^1 \int_{-x^2}^{x^2} 3 - 2x^2 + y \, dy \, dx.$$

(c) Evaluating gives us a volume

$$
\begin{aligned}
V = \int_0^1 \int_{-x^2}^{x^2} 3 - 2x^2 + y \, dy \, dx &= \int_0^1 3y - 2x^2 y + \frac{1}{2}y^2 \Big|_{-x^2}^{x^2} dx \\
&= \int_0^1 3x^2 - 2x^4 + \frac{1}{2}x^4 - \left(-3x^2 + 2x^4 + \frac{1}{2}x^4 \right) dx \\
&= \int_0^1 6x^2 - 4x^4 \, dx \\
&= 2x^3 - \frac{4}{5}x^5 \Big|_0^1 \\
&= 2 - \frac{4}{5}.
\end{aligned}
$$

(d) Good luck on your finals! I hope you all enjoyed class at least a little bit this quarter! ☺