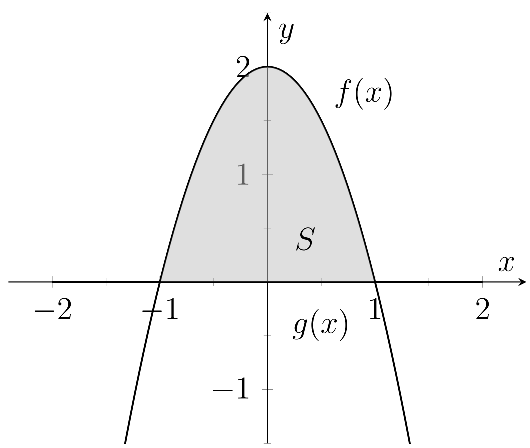
Note: The best way to start a volume problem is to draw pictures.

Volume Formulae	**Explanation**
$Volume = \int_a^b A(x)\, dx$	General rule for finding volume of a solid with respect to x. $A(x)$ is the area of the cross section (at position x).
$Volume = \int_c^d A(y)\, dy$	General rule for finding volume of a solid with respect to y. $A(y)$ is the area of the cross section (at position y).

How to Find $A(x)$	**Explanation**
$A(x) = \pi(R^2 - r^2)$	If the cross sections are disks or washers. $R =$ outer radius $r =$ inner radius
$A(x) = 2\pi r \cdot h$	If the cross sections are cylinders. $r =$ radius of cylinder $h =$ height

1. Let R be the region bounded by the graphs of $f(x) = 2 - 2x^2$ and $g(x) = 0$.
   (a) Sketch the region bounded by $f(x)$ and $g(x)$ on the same coordinate plane, and label the region S. Find the intersection points.

Solution:



The intersection points are just at the roots of f:

$$2 - 2x^2 = 0$$

gives

$$x = -1, \qquad x = 1.$$

(b) Set up an integral to find the volume of the solid formed by revolving the region S about the x-axis. **Do not evaluate the integral.**

Solution:

$$\text{Volume } = \pi \int_{-1}^{1} \left(2 - 2x^2\right)^2 \; dx$$

(c) Set up an integral to find the volume of the solid formed by revolving the region S about the line $y = 5$. **Do not evaluate the integral.**

Solution:

$$\text{Volume } = \int_{-1}^{1} \pi 5^2 - \pi \left[5 - (2 - 2x^2)\right]^2 \; dx = \pi \int_{-1}^{1} 25 - \left(3 + 2x^2\right)^2 \; dx$$

(d) Set up an integral to find the volume of the solid formed by revolving the region S about the line $x = -2$. **Do not evaluate the integral.** (Hint: Cylinders)

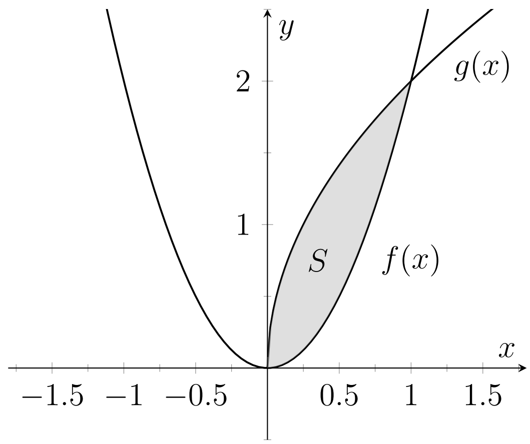
Solution:

$$\text{Volume } = \int_{-1}^{1} 2\pi(x - (-2)) \left(2 - 2x^2\right) \; dx = 2\pi \int_{-1}^{1} (x + 2) \left(2 - 2x^2\right) \; dx$$

2. Let S be the region bounded by the graphs of $f(x) = 2x^2$ and $g(x) = 2\sqrt{x}$. Solve the following problems.

(a) Sketch the region, and label it S. Find its intersection points

Solution:



We find the intersection points by setting the two equal:

$$2x^2 = 2\sqrt{x}.$$

Solving gives us

$$x = 0, \qquad x = 1.$$

(b) Set up an integral to find the volume of the solid formed by revolving the region S about the y-axis. **Do not evaluate the integral.**

Solution:

$$\begin{aligned} \text{Volume} &= \int_0^2 \pi \left(\sqrt{\frac{y}{2}}\right)^2 - \pi \left(\left(\frac{y}{2}\right)^2\right)^2 \, dy = \pi \int_0^2 \frac{y}{2} - \frac{y^4}{16}\, dy && \text{(washers)} \\ &= \int_0^1 2\pi(x)\left(2\sqrt{x} - 2x^2\right)\, dx = 4\pi \int_0^1 x^{3/2} - x^3\, dx && \text{(cylinders)} \end{aligned}$$

(c) Set up an integral to find the volume of the solid formed by revolving the region S about the line $x = -1$. **Do not evaluate the integral.**

Solution:

$$\begin{aligned} \text{Volume} &= \int_0^2 \pi\left(\sqrt{\frac{y}{2}}+1\right)^2 - \pi\left(\left(\frac{y}{2}\right)^2+1\right)^2\,dy && \text{(washers)} \\ &= \int_0^1 2\pi(x+1)\left(2\sqrt{x}-2x^2\right)\,dx && \text{(cylinders)} \end{aligned}$$

3. Sketch the region S bounded by the curves $y = 4 - x^2$ and $y = x^2 - 4$. Find the volume of the solid obtained by rotating R about the line $x = 2$.