

1. Consider the following system of linear equations.

$$x + 2y + 3z = 0$$

$$4x + 5y + 6z = 0$$

$$7x + 8y + 9z = 0$$

(a) Find the augmented matrix of the system.

$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 0 \end{array} \right]$$

(b) Find the reduced row echelon form of the matrix.

$$\left[ \begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 0 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow R_2 - 4R_1 \\ R_3 \rightarrow R_3 - 7R_1}} \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \\ 0 & -6 & -12 & 0 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow -\frac{1}{3}R_2 \\ R_3 \rightarrow -\frac{1}{6}R_3}} \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array} \right]$$

$$\xrightarrow{R_3 \rightarrow R_3 - R_2} \left[ \begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left[ \begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

(c) What is the general solution to the system?

$$\text{From (b), } \left. \begin{array}{l} x - z = 0 \\ y + 2z = 0 \end{array} \right\} \Rightarrow \begin{array}{l} x = z \\ y = -2z \end{array}$$

and  $z$  is free.

$$\therefore \text{General soln is } (x, y, z) = (z, -2z, z)$$

$$\text{These are perfect too: } \begin{array}{l} x = z \\ y = -2z \\ z = z \end{array} \quad \text{OR} \quad \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} z$$