MATH 4A | QUIZ 2 | WINTER, 2018

Name: Solns. TA Name: You should know this Section Time: MW

Please show all your work! Answers without supporting work will not be given credit.
Each question is worth 15 points.

1. Complete the following definition:

   A **subspace** of $\mathbb{R}^n$ is any set H in $\mathbb{R}^n$ that has the following three properties:

   (a) $\vec{0} \in H$

   (b) If $\vec{u}, \vec{v} \in H$, then $\vec{u} + \vec{v} \in H$.

   (c) If $u \in H$, then $c\vec{u} \in H$, where c is ANY scalar.

Call this subset H.

2. Determine if the subset of $\mathbb{R}^3$ consisting of all vectors of the form $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where $a - b = c$ is a subspace.

   If it is a subspace, prove your claim. If it is not a subspace, show which property it violates and give a counterexample.

$\boxed{\text{Yes}}$ **(a)** Since $0 - 0 = 0$, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in H$.

(b) Let $\vec{u}, \vec{v} \in H$. Then, $\vec{u} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$ with $a_1 - b_1 = c_1$ and $\vec{v} = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$ with $a_2 - b_2 = c_2$. Since $\vec{u} + \vec{v} = \begin{bmatrix} a_1 + a_2 \\ b_1 + b_2 \\ c_1 + c_2 \end{bmatrix}$ and

$(a_1 + a_2) - (b_1 + b_2) = (a_1 - b_1) + (a_2 - b_2) = c_1 + c_2$, $\vec{u} + \vec{v} \in H$.

(c) Let $\vec{u} \in H$. Then, $\vec{u} = \begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix}$ with $a_1 - b_1 = c_1$.

Let k be a scalar. Then, $k\vec{u} = \begin{bmatrix} ka_1 \\ kb_1 \\ kc_1 \end{bmatrix}$ and

$ka_1 - kb_1 = k(a_1 - b_1) = kc_1$. Hence, $k\vec{u} \in H$.

Since all properties are satisfied, H is a subspace. ∎