MATH 4A

Quiz 3 - Monday, March 5

Name: Solns

Section Time: MW

Problem 1: Given the matrices A and B below, use determinants to determine if the following matrices are invertible. If so, find the inverse.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & 1 \\ 0 & -2 & -2 \\ 3 & 11 & 5 \end{bmatrix}$$

- A $\det(A) = 1 \cdot 1 \cdot 2 = 2$, so **A is invertible**

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 \to 1/2\, R_3]{R_1 \to R_1 - 2R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right] \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 + R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -2 & 1/2 \\ 0 & 1 & 0 & 0 & 1 & -1/2 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right].$$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1/2 \end{bmatrix}$$

- B

$$\det(B) = 1 \cdot \begin{vmatrix} -2 & -2 \\ 11 & 5 \end{vmatrix} + 3 \begin{vmatrix} 3 & 1 \\ -2 & -2 \end{vmatrix} = \overbrace{(-10 + 22)}^{12} + 3\overbrace{(-6 + 2)}^{(-4)} = 0.$$

$\det(B) = 0$, so **B is NOT invertible**

- AB

Since $\det(AB) = \det(A) \cdot \underbrace{\det(B)}_{0} = 0$,

AB is also not invertible

alternatively $\begin{vmatrix} 1 & 3 & 1 \\ 0 & -2 & -2 \\ 3 & 11 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 1 \\ 0 & -2 & -2 \\ 0 & 2 & 2 \end{vmatrix}$ because $R_3 \to R_3 - 3R_1$

Now, $\det(B) = 1 \begin{vmatrix} -2 & -2 \\ 2 & 2 \end{vmatrix} - 0 + 0 = -2(2) - (-2)(2) = 0$