THIS WEEK. (REVIEW of BASIS / DIMENSION / NULL(A) / Range)

LET $A = \begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 3 & 4 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

(1) FIND dim NULL(A)

dim Range(A) = SPACE span by THE COLUMNS of A = COL(A)

FIND A BASIS of NULL(A)

A BASIS of Range(A).

(2) $\begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 3 & 4 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix}_{3\times 4} : \mathbb{R}^4 \longrightarrow \mathbb{R}^3$

$NULL(A) = \left\{ \begin{pmatrix} x \\ y \\ w \\ z \end{pmatrix} \in \mathbb{R}^4 \;/\; A\begin{pmatrix} x \\ y \\ w \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

$Range(A) = \left\{ \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in \mathbb{R}^3 \;/\; A\begin{pmatrix} x \\ y \\ w \\ z \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \text{ has solution} \right\}$.

$$\begin{pmatrix} 1 & 2 & 2 & 1 \\ 2 & 3 & 4 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 2 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} \underline{1} & 2 & 2 & 1 \\ 0 & \underline{-1} & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ PIVOTS.

THUS THE FIRST TWO COLUMNS of A are linearly independent (THE OTHER ARE FREE VARIABLES)

So

dim SPAN of COLUMN of $A \equiv 2$

A basis of SPAN of COLUMNS of $A = \begin{pmatrix}1\\2\\0\end{pmatrix}\begin{pmatrix}2\\3\\1\end{pmatrix}$

$$\mathrm{NULL}(A) = \left\{ \begin{pmatrix}x\\y\\w\\z\end{pmatrix} \in \mathbb{R}^4 \;/\; A\begin{pmatrix}x\\y\\w\\z\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} \right\}$$

$$\begin{pmatrix} 1 & 2 & 2 & 1 & 0 \\ 2 & 3 & 4 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & 2 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

free variables w, z

$$\begin{cases} x + 2y + 2w + z = 0 \\ 2x + 3y + 4w + z = 0 \\ y + z = 0 \end{cases}$$

$$\begin{cases} x + 2y = -2w - z \\ 2x + 3y = -4w - z \\ y = -z \end{cases} \Rightarrow$$

z, w free

$y = -z$

$x = -2y - 2w - z = z - 2w$

THUS

$$\begin{pmatrix}x\\y\\w\\z\end{pmatrix} = \begin{pmatrix}z-2w\\-z\\w\\z\end{pmatrix} = w\begin{pmatrix}-2\\0\\1\\0\end{pmatrix} + z\begin{pmatrix}1\\-1\\0\\1\end{pmatrix} \in \mathrm{NULL}(A)$$

$\begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$ linearly independent

$$\mathrm{NULL}(A) = \mathrm{span} \begin{pmatrix} -2 \\ 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\dim(\mathrm{NULL}(A)) = 2.$$