## PRODUCT of MATRICES.

$A_{m \times n}$ $B_{n \times q}$ matrices

m - rows n - rows
n - columns q - columns

$A \cdot B$ is an $m \times q$ matrix.

EX $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 0 \end{pmatrix}_{3\times 2}$ $B = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}_{2\times 2}$

$$A \cdot B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1\cdot 1 + 2\cdot 2 & 1\cdot(-1) + 2\cdot 0 \\ 3\cdot 1 + 4\cdot 2 & 3\cdot(-1) + 4\cdot 0 \\ 5\cdot 1 + 0\cdot 2 & 5\cdot(-1) + 0\cdot 0 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 11 & -3 \\ 5 & 5 \end{pmatrix}_{3\times 2}$$

In general even if $A, B$ are $n \times n$ (square) matrices in general

$$AB \neq BA.$$

$$A = \begin{pmatrix} a & 0 \\ 1 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 \\ 3 & b \end{pmatrix} \quad \text{any } a, b \in \mathbb{R}.$$

$$AB = \begin{pmatrix} a & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & b \end{pmatrix} = \begin{pmatrix} a & 2a \\ 7 & 2+2b \end{pmatrix}$$

$$BA = \begin{pmatrix} 1 & 2 \\ 3 & b \end{pmatrix}\begin{pmatrix} a & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} a+2 & 4 \\ 3a+b & 2b \end{pmatrix} \neq AB$$

Let $A$ be an $n \times n$ matrix if there exists $B$ $n \times n$ matrix st. $AB = BA = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} = I$

We say that $A$ is invertible and $B = A^{-1}$.

## EX

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 3/5 & -2/5 \\ 1/5 & 1/5 \end{pmatrix} = A^{-1} : AB = BA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

**APPLICATION** SOLVE $\underbrace{A \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ -15 \end{pmatrix}}_{*}$

$$\begin{cases} x + 2y = 10 \\ -x + 3y = -15 \end{cases}$$

MULTIPLYING $*$ by $A^{-1}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3/5 & -2/5 \\ 1/5 & 1/5 \end{pmatrix} \begin{pmatrix} 10 \\ -15 \end{pmatrix} = \begin{pmatrix} 12 \\ -1 \end{pmatrix}$$

Lpd.