

MATH 4A Section = (#2)

①

FIND ALL SOLUTIONS OF THE HOMOGENEOUS SYSTEM

$$(1) \begin{cases} x_1 + 2x_2 + 3x_3 - 2x_4 = 0 \\ 2x_1 + 4x_2 + 8x_3 + x_4 = 0 \\ 3x_1 + 6x_2 + 13x_3 + 4x_4 = 0 \end{cases}$$

$$A = \begin{pmatrix} 1 & 2 & 3 & -2 & : & 0 \\ 2 & 4 & 8 & 1 & : & 0 \\ 3 & 6 & 13 & 4 & : & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 3 & -2 & : & 0 \\ 0 & 0 & 2 & 5 & : & 0 \\ 0 & 0 & 4 & 10 & : & 0 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 3 & -2 & : & 0 \\ 0 & 0 & \frac{2}{2} & 5 & : & 0 \\ 0 & 0 & 0 & 0 & : & 0 \end{pmatrix}$$

↑ ↑
free variables

$$\begin{cases} x_1 + 2x_2 + 3x_3 - 2x_4 = 0 \\ 2x_3 + 5x_4 = 0 \end{cases}$$

$$\Rightarrow \boxed{x_3 = -\frac{5}{2}x_4}$$

$$x_1 = -2x_2 - 3x_3 + 2x_4$$

$$= -2x_2 + \frac{15}{2}x_4 + 2x_4 = -2x_2 + \frac{19}{2}x_4$$

THUS

$$\boxed{x_1 = -2x_2 + \frac{19}{2}x_4}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2x_2 + \frac{19}{2}x_4 \\ x_2 \\ -\frac{5}{2}x_4 \\ x_4 \end{pmatrix}$$

$$= x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} 19/2 \\ 0 \\ -5/4 \\ 1 \end{pmatrix}$$

THUS:

THE SET OF SOLUTIONS OF THE HOMOGENEOUS SYSTEM (1) IS \equiv

$$\text{SPAN} \{ \vec{v}_1, \vec{v}_2 \}, \quad \vec{v}_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} 19/2 \\ 0 \\ -5/4 \\ 1 \end{pmatrix}$$

Prove that $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 4 \\ 9 \\ 5 \end{pmatrix}$

are LINEARLY INDEPENDENT.

Prove that $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 4 \\ 5 \\ 5 \end{pmatrix}$

are LINEARLY DEPENDENT.

ANSWER QUESTIONS

THANKS!