Math 8
Worksheet
Week 1, Thursday

Implication and Proof

Collaborators:

Often, we wish to show that some **conditional statement** is or is not true. Suppose we have statments P and Q. We may introduce the conditional statement

$$P \implies Q,$$

read "P implies Q". As it sounds, this statement asserts "if P, then Q". That is, if the statement P is true, then the statement Q must also be true. It is left to us to then evaluate whether this implication is valid or not.

The **converse** of a conditional statement $P \implies Q$ is the conditional statement $Q \implies P$. If $P \implies Q$ and $Q \implies P$, then we write $P \iff Q$ and say "P if and only if Q".

We may also consider the opposite of a statement, referred to as the **negation**.

The **contrapositive** of a conditional statement $P \implies Q$ is the conditional statement $\overline{Q} \implies \overline{P}$.

Prove the following or give a counterexample.

a) If n is an even integer, then n^2 is an even integer.

b) If n is an integer such that n^2 is even, then n is even.

c) If n is an integer such that $n^2 + 3$ is odd, then n is odd.

d) If $n = m^3 - m$ for some integer m, then n is a multiple of 6.

Scratch Work

Proof.

□

One way to show $P \implies Q$ is to assume both P and $\overline{Q}$. If we reach a contradiction, a statement that is clearly false, we see that we cannot simultaneously have P and $\overline{Q}$. Thus, if we have P, then we must have Q as well.

Prove by contradiction that a real number that is less than every positive real number cannot be positive.

Scratch Work

Proof.

$\square$

Homework problem: Write down a careful proof of the following statement:

$$\sqrt{6} - \sqrt{2} > 1.$$

Scratch Work

Proof.

□