Math 8
Worksheet
Week 5, Thursday

Fundamental Theorem of Arithmetic

Collaborators:

Here we restrict our attention to the integers. Given integers a and b, we say "a divides b" or "b is divisible by a", and we write $a \mid b$, if there exists another integer c such that $ac = b$. An integer p is called **prime** if $a \mid p$ implies $a = \pm 1$ or $a = \pm p$. Since negative numbers do not significantly add anything interesting to the theory of divisibility, we typically restrict our attention further to nonnegative integers.

HW: Liebeck Chapter 10, Problem 4(b) Suppose a, b are integers such that $a \mid b$ and $b \mid a$. Prove that $a = \pm b$.

Theorem (Fundamental Theorem of Arithmetic)**.** *Let $n \geq 2$ be an integer.*

(a) Then n is equal to a product of prime numbers,

$$n = p_1 \cdots p_k,$$

where $p_1, \ldots, p_k$ are prime and $p_1 \leq p_2 \leq \cdots \leq p_k$.

*(b) This **prime factorization** is unique. That is, if*

$$n = p_1 \cdots p_k = q_1 \cdots q_\ell$$

with p_i, q_i prime as in part (a), then $k = \ell$ and $p_i = q_i$ for all i.

Let $n = p_1^{a_1} \cdots p_k^{a_k}$, where the p_i are prime with $p_1 < \cdots < p_k$ and the a_i are positive integers. Show that if $m \mid n$, then

$$m = p_1^{b_1} \cdots p_k^{b_k}$$

with $0 \leq b_i \leq a_i$ for all i.