Worksheet 1
Smooth Spaces

Determine the dimension of, and describe a system of coordinates for, each of the following spaces $M$.

1. $M = \text{line segments in } \mathbb{R}^1$
2. $M = \text{rays in } \mathbb{R}^2$
3. $M = \text{circles in } \mathbb{R}^2$
4. $M = \text{squares in } \mathbb{R}^2$
5. $M = \text{lines in } \mathbb{R}^2$
6. $M = \text{line segments in } \mathbb{R}^2$
7. $M = \text{points in } \mathbb{R}^3$
8. $M = \text{spheres in } \mathbb{R}^3$
9. $M = \text{planes in } \mathbb{R}^3$
10. $M = \text{circles in } \mathbb{R}^3$

11. Look up the six “lower kinematic pairs”. Determine the dimension of, and describe a system of coordinates for, the configuration space of each one.

12. Determine the dimension, and describe a system of coordinates for, the space of colors. (*Hint:* Look up the “RGB color model”.)

13. Look up “gimbal lock”. Describe this phenomenon in terms of the dimension of the configuration space of a gyroscope.

14. Determine the dimension of, and describe a system of coordinates for, the following families of probability distributions:
   (i) Bernoulli
   (ii) Normal
   (iii) Poisson

15. Determine the dimension of, and describe a system of coordinates for, the space of musical pitches.