Find and sketch the critical points of the following functions $f : N \to \mathbb{R}$. Try to make an educated guess about whether each critical point is a local minimum, local maximum, or neither.

1. $N = \text{the unit circle in } \mathbb{R}^2$. $f(p) = \text{the square of the distance from } p \text{ to the point } \begin{pmatrix} 0 \\ 2 \end{pmatrix} \in \mathbb{R}^2$.

2. $N = \text{the unit circle in } \mathbb{R}^2$. $f(p) = \text{the square of the distance from } p \text{ to } \begin{pmatrix} 0 \\ 0 \end{pmatrix} \in \mathbb{R}^2$.

3. $N = \text{the unit sphere in } \mathbb{R}^3$. $f(p) = \text{the square of the distance from } p \text{ to } \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} \in \mathbb{R}^3$.

4. $N = \text{the space of linear functions } p(x) = ax + b \text{ which satisfy } \int_0^1 p(x) \, dx = 0$. $f(p) = \int_0^1 p(x)^2 \, dx$.

5. $N = \text{the space of functions } p : \{1, 2\} \to \mathbb{R} \text{ with } p(1) + p(2) = 1$. $f(p) = 1 \cdot p(1) + 2 \cdot p(2)$, the expected value of $p$.

6. Fix a natural number $n \in \mathbb{N}$ and let $N = \text{the space of functions } p : \{1, 2, \ldots, n\} \to \mathbb{R} \text{ which satisfy } \sum_{k=1}^n p(k) = 1$. $f(p) = \sum_{k=1}^n k \cdot p(k)$.

7. $N = \text{the space of cardboard boxes of volume } 1 \text{ cubic meter}$. $f(p) = \text{the surface area of } p$.

8. $N = \text{the space of cylindrical containers of volume } 1 \text{ cubic foot}$. $f(p) = \text{the surface area of } p$.

9. You have 10 square meters of wood with which to build a rectangular shelter, that is, a box with four sides, a top, but no bottom. What is the greatest volume your shelter can contain?

10. You would like to build a four-sided wooden garden planter of volume 1 cubic meter. Each side of the planter should have equal area. What is the least amount of wood you need to buy?