Effective dynamics for ferromagnetic thin films

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In a ferromagnetic material, the dynamics of the relaxation process are affected by the presence of a strong shape or material anisotropy. In this article, we systematically explore this fact to derive the effective dynamical equation for a soft ferromagnetic thin film. We show that, as a consequence of the interplay between shape anisotropy and damping, the gyromagnetic term is effectively also a damping term for the in-plane components of the magnetization distribution. We validate our result through numerical simulation of the original Landau–Lifshitz equation and our effective equation.
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INTRODUCTION

The dynamics of the magnetization distribution in a ferromagnetic thin film are an interesting and important problem from both scientific and technological points of view. Improvement of deposition (lithography) techniques allows such films to be made with high precision and relative ease. Interest in using them as magnetic memory devices (MRAMs) has given a greater incentive to study this subject. Since defects, impurities, and thermal noise play important roles in the dynamics of the magnetization field in nanometer thick films, it also makes an ideal playground for studying some of the nanoscale physics.[1–5]

The dynamics of the magnetization distribution in a ferromagnetic material are described by the Landau–Lifshitz equation,[6,7]

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M}\times\boldsymbol{\mathcal{H}} - \frac{\gamma\alpha}{M_s}\mathbf{M}\times\mathbf{M}\times\boldsymbol{\mathcal{H}}, \tag{1}$$

where $|\mathbf{M}|=M_s$ is the saturation magnetization, and is usually set to be a constant far from the Curie temperature; γ is the gyromagnetic ratio and is given by $\gamma=ge/(2m_e)$, where e and m_e are the (positive) charge and mass of the electron, respectively, and g has values close to 2 for many ferromagnetic materials. The first term on the right-hand side is the gyromagnetic term and the second term is the damping term. α is the dimensionless damping coefficient. $\boldsymbol{\mathcal{H}}$ is the local field, computed from the Landau–Lifshitz free energy functional:

$$\boldsymbol{\mathcal{H}} = -\frac{\delta F}{\delta \mathbf{M}}, \tag{2}$$

$$F[\mathbf{M}] = \frac{1}{2}\int_\Omega \left\{ \Phi\left(\frac{\mathbf{M}}{M_s}\right) + \frac{C_{\mathrm{ex}}}{M_s^2}|\nabla \mathbf{M}|^2 - 2\mu_0 \mathbf{H}_e\cdot\mathbf{M}\right\} dx + \frac{\mu_0}{2}\int_{\mathbb{R}^3}|\nabla U|^2\, dx. \tag{3}$$

In Eq. (3), C_{ex} is the exchange constant, $C_{\mathrm{ex}}|\nabla M|^2/M_s^2$ is the exchange interaction energy between the spins, $\Phi(\mathbf{M}/M_s)$ is the energy due to material anisotropy, μ_0 is the permeability of vacuum, $-2\mu_0\mathbf{H}_e\cdot\mathbf{M}$ is the energy due to the external applied field, Ω is the volume occupied by the material, and finally the last term in Eq. (3) is the energy due to the field induced by the magnetization distribution inside the material. This induced field $\mathbf{H}_s = -\nabla U$ can be computed by solving

$$\Delta U = \begin{cases} \nabla\cdot\mathbf{M} & \text{in } \Omega, \\ 0 & \text{outside } \Omega, \end{cases} \tag{4}$$

together with the jump conditions

$$[U]=0, \tag{5}$$

$$\left[\frac{\partial U}{\partial \nu}\right] = -\mathbf{M}\cdot\nu, \tag{6}$$

at the material–vacuum interface. In Eqs. (5) and (6) we denote by [ ] the jump of a quantity across the interface. The vector ν represents the outward unit normal on the boundary of Ω.

Since $\mathbf{M}$ and $\boldsymbol{\mathcal{H}}$ have the same physical dimensions, we can write $\boldsymbol{\mathcal{H}}=M_s\mathbf{h}$, $\mathbf{H}_s=M_s\mathbf{h}_s$, $\mathbf{H}_e=M_s\mathbf{h}_e$, and $\mathbf{M}=M_s\mathbf{m}$. Without loss of generality, we will assume that the material is uniaxial, and that $\Phi(\mathbf{m})=K_u(m_2^2+m_3^2)$. Equation (1) is rewritten as

$$\frac{\partial \mathbf{m}}{\partial t} = -\mu_0\gamma M_s\ \mathbf{m}\times\mathbf{h} - \mu_0\gamma M_s\alpha\ \mathbf{m}\times\mathbf{m}\times\mathbf{h}, \tag{7}$$

where

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$$\mathbf{h}=-\frac{K_u}{\mu_0 M_s^2}(m_2\mathbf{e}_2+m_3\mathbf{e}_3)+\frac{C_{\rm ex}}{\mu_0 M_s^2}\Delta\mathbf{m}+\mathbf{h}_s+\mathbf{h}_e. \tag{8}$$

Here we use the notation $\mathbf{e}_1=(1,0,0)$, $\mathbf{e}_2=(0,1,0)$, and $\mathbf{e}_3=(0,0,1)$.

The constant $\mu_0\gamma M_s$ has dimensions of the reciprocal of time (s^{-1}). Therefore we rescale in time: $t\rightarrow(\mu_0\gamma M_s)^{-1}t$, and we rescale the spatial variable $x\rightarrow Lx$ where L is the diameter of Ω. The equation becomes

$$\frac{\partial\mathbf{m}}{\partial t}=-\mathbf{m}\times\mathbf{h}-\alpha\,\mathbf{m}\times\mathbf{m}\times\mathbf{h}, \tag{9}$$

where

$$\mathbf{h}=-Q(m_2\mathbf{e}_2+m_3\mathbf{e}_3)+\eta\Delta\mathbf{m}+\mathbf{h}_s+\mathbf{h}_e. \tag{10}$$

Here we have defined the dimensionless parameters $Q=K_u/(\mu_0 M_s^2)$, and $\eta=C_{\rm ex}/(\mu_0 M_s^2 L^2)$.

RESULTS AND DISCUSSION

In many applications, the material is a thin film. Assume that the lateral dimensions of the material occupy the xy plane and the thickness is δ in dimensionless units ($\delta\ll 1$). For thin films, the energy due to surface charges is more significant than the energy due to volume charges ($=\nabla\cdot\mathbf{m}$). Then, from the viewpoint of energetics, the z component of $\mathbf{m}$, m_3, must be small since it contributes to the surface charge. As a result, the leading order energetics in a thin film is much simplified.

To be more precise, let us assume that $\mathbf{m}$ does not change across the film. We will write $\mathbf{m}=(\mathbf{m}',m_3)$, where $\mathbf{m}'=(m_1,m_2)$, and use $\hat{f}$ to denote the Fourier transform of f.

The energy of the self-induced field can be written in Fourier space as[8,9]

$$E_M=\frac{\mu_0\delta}{2}\int_{\mathbb{R}^2}\frac{(\xi\cdot\hat{\mathbf{m}}')^2}{|\xi|^2}(1-\Gamma_\delta(|\xi|))\,d\xi$$
$$+\frac{\mu_0\delta}{2}\int_{\mathbb{R}^2}\hat{m}_3^2\Gamma_\delta(|\xi|)\,d\xi, \tag{11}$$

where

$$\xi=(\xi_1,\xi_2)\quad\text{and}\quad\Gamma_\delta(|\xi|)=\frac{1-e^{-2\pi\delta|\xi|}}{2\pi\delta|\xi|}. \tag{12}$$

For $|\xi|\ll 1/\delta$, E_M simplifies to

$$E_M=\frac{\mu_0\pi\delta^2}{2}\int_{\mathbb{R}^2}\frac{(\xi\cdot\hat{\mathbf{m}}')^2}{|\xi|}d\xi+\frac{\mu_0\pi\delta}{2}\int_{\mathbb{R}^2}\hat{m}_3^2\,d\xi. \tag{13}$$

Since the first term on the right-hand side contains an extra small coefficient δ, m_3 must be small compared with (m_1,m_2). For a more thorough treatment of the energetics in thin films, we refer the reader to Refs. 10 and 11.

The dynamics in a thin film are also severely constrained. As an indication, we note that, from Eq. (13), the self-induced field takes the form

$$\mathbf{h}_s=(\delta L_1(m_1,m_2),\delta L_2(m_1,m_2),m_3). \tag{14}$$

This motivates us to introduce the rescaling $m_3\rightarrow\delta\tilde{m}_3$, $t\rightarrow\delta\tilde{t}$, $h_1\rightarrow\delta\tilde{h}_1$, and $h_2\rightarrow\delta\tilde{h}_2$. By using this rescaling, we implicitly assume that both parameters Q and η, and the external field $\mathbf{h}_e$, are small: $Q\sim\delta$, $\eta\sim\delta$, and $|\mathbf{h}_e|\sim\delta$. The Landau–Lifshitz system of equations can therefore be rewritten as

$$\delta\frac{\partial m_1}{\partial\tilde{t}}=\delta^2\tilde{m}_3\tilde{h}_2-\delta m_2\tilde{h}_3+\alpha(\delta\tilde{h}_1-\delta(\mathbf{m}'\cdot\tilde{\mathbf{h}}')m_1$$
$$-\delta^2\tilde{m}_3\tilde{h}_3 m_1),$$

$$\delta\frac{\partial m_2}{\partial\tilde{t}}=\delta m_1\tilde{h}_3-\delta^2\tilde{m}_3\tilde{h}_1+\alpha(\delta\tilde{h}_2-\delta(\mathbf{m}'\cdot\tilde{\mathbf{h}}')m_2$$
$$-\delta^2\tilde{m}_3\tilde{h}_3 m_2),$$

$$\delta^2\frac{\partial m_3}{\partial\tilde{t}}=\delta m_2\tilde{h}_1-\delta m_1\tilde{h}_2+\alpha(\delta\tilde{h}_3-\delta^2(\mathbf{m}'\cdot\tilde{\mathbf{h}}')\tilde{m}_3$$
$$-\delta^3\tilde{m}_3\tilde{h}_3\tilde{m}_3).$$

Collecting the leading order terms, assuming that $\delta\ll\alpha$, we get

$$\tilde{h}_3=\frac{1}{\alpha}(m_1\tilde{h}_2-m_2\tilde{h}_1), \tag{15}$$

$$\begin{aligned}\frac{\partial m_1}{\partial\tilde{t}}&=-m_2\tilde{h}_3+\alpha(\tilde{h}_1-(\mathbf{m}'\cdot\tilde{\mathbf{h}}')m_1),\\ \frac{\partial m_2}{\partial\tilde{t}}&=m_1\tilde{h}_3+\alpha(\tilde{h}_2-(\mathbf{m}'\cdot\tilde{\mathbf{h}}')m_2).\end{aligned} \tag{16}$$

To leading order, $m_1^2+m_2^2=1$, so we can write

$$\begin{aligned}m_2\tilde{h}_3&=\frac{1}{\alpha}m_2(m_1\tilde{h}_2-m_2\tilde{h}_1)=-\frac{1}{\alpha}(\tilde{h}_1-(\mathbf{m}'\cdot\tilde{\mathbf{h}}')m_1),\\ m_1\tilde{h}_3&=\frac{1}{\alpha}(\tilde{h}_2-(\mathbf{m}'\cdot\tilde{\mathbf{h}}')m_2).\end{aligned} \tag{17}$$

Thus, the third component can be effectively eliminated from the equations.

Going back to the time scale t, we have

$$\begin{aligned}\frac{\partial m_1}{\partial t}&=\delta\left(\frac{1}{\alpha}+\alpha\right)(\tilde{h}_1-(m_1\tilde{h}_1+m_2\tilde{h}_2)m_1),\\ \frac{\partial m_2}{\partial t}&=\delta\left(\frac{1}{\alpha}+\alpha\right)(\tilde{h}_2-(m_1\tilde{h}_1+m_2\tilde{h}_2)m_2).\end{aligned} \tag{18}$$

Equation (18) is the gradient flow of the in-plane components $\mathbf{m}'=(m_1,m_2,0)$ associated with the reduced energy

$$F[\mathbf{m}']=\frac{1}{2}\int_{\Omega'}\{C_{\rm ex}|\nabla\mathbf{m}'|^2+\Phi(\mathbf{m}')-2\mathbf{h}_e\cdot\mathbf{m}'\}\,dx$$
$$+\frac{1}{2}\int_{\mathbb{R}^3}|\nabla u|^2\,dx. \tag{19}$$

This analysis reveals several interesting facts about thin films. Equation (15) indicates that, in a thin film, the dynamics of the normal component of the magnetization, m_3, are slaved by the in-plane components. As a result, the gyromagnetic term is effectively also a dissipative term for the in-plane components. Equation (18) shows the seemingly paradoxical fact that the dynamics in very thin films can be accelerated by decreasing the value of the damping coefficient α. One of the advantages of our effective Eq. (18) is that it is much simpler to solve numerically than the full Landau–Lifshitz equation. The time stepping can be performed with the methods described in Refs. 12 and 13. The complexity of these methods is comparable to that of solving scalar heat equations implicitly, and the time step size required for stability is independent of the grid size.

Going back to the general situation described by the original Landau–Lifshitz equation (1), the first term on the right-hand side is the conservative gyromagnetic term. The second term is the damping term. Typically $\alpha \ll 1$ ($\alpha \approx 0.01-0.1$) so the conservative term dominates. Our previous argument suggests that the effective dynamic equation (18) is valid when $\delta \ll \alpha$. In that case, the small dissipative term triggers a large dissipative effect from the conservative gyromagnetic term (with coefficient $1/\alpha$) for the in-plane components because of the severe geometric constraint. However, for typical samples used in MRAM applications, we often have $\delta \approx \alpha$.

NUMERICAL EXPERIMENTS

In order to assess the validity of our effective equation (18) in different parameter regimes, we carried out direct micromagnetics simulations of the Landau–Lifshitz equations. We used the following values of the parameters to model a permalloy film:

$$\mu_0 = 4\pi \times 10^{-7} \ \ \mathrm{N/A^2},$$

$$\gamma = 1.76 \times 10^{11} \ \ \mathrm{T^{-1}\,s^{-1}},$$

$$M_s = 8.0 \times 10^5 \ \ \mathrm{A/m},$$

$$K_u = 5.0 \times 10^2 \ \ \mathrm{J/m^3},$$

$$C_{\mathrm{ex}} = 1.3 \times 10^{-11} \ \ \mathrm{J/m}.$$

We compute the effective field by approximating the magnetization using piecewise bilinear functions. For the time stepping we have used a fourth order Runge–Kutta method. In our simulations, we have used 128 and 256 grid points in each direction, which is enough to resolve the domain walls.

In our first experiment we compare the solution to the equations for different values of the aspect ratio, δ, fixing α at 0.1. A sketch of the initial and final configuration is shown in Fig. 1. In Figs. 2 and 3 we present images at different times during the relaxation process with and without an external field. In these images, we show the divergence of the in-plane components (m_1, m_2) on a gray scale. The movies are available online, and can be viewed at http://www.math.princeton.edu/~cgarcia. The first and third columns are the results of the full simulation of the original Landau–Lifshitz equation, and the second and fourth columns are the results of our effective equation. The results for the Landau–Lifshitz equation and our effective equation for an aspect ratio $\delta = 2.\times 10^{-2}$ are presented in the first two columns for comparison. In the other two columns we compare the results for an aspect ratio $\delta = 1.25 \times 10^{-3}$ (16 times smaller). We clearly see the presence of fast moving spin waves in the simulations of the full Landau–Lifshitz equation for the two values of the aspect ratio. These spin waves are more evident when $\delta = 2.\times 10^{-2}$. In the effective equation there are no such waves, but the overall relaxational dynamics for both values of δ seem to be correctly captured by Eq. (18). Certainly closer agreement is found for the smaller value of δ. Therefore, the effective dynamics do

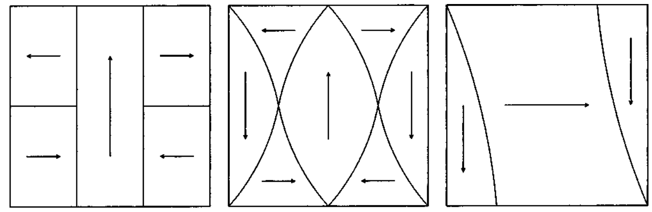

FIG. 1. Initial and final configurations in Figs. 2 and 3. The picture on the left is the initial configuration. The picture in the middle is the final configuration when there is no applied field. The picture on the right is the final configuration when there is an applied field $\mathbf{h}_e = 2\delta[\cos(\pi/180), \sin(\pi/180), 0]$.

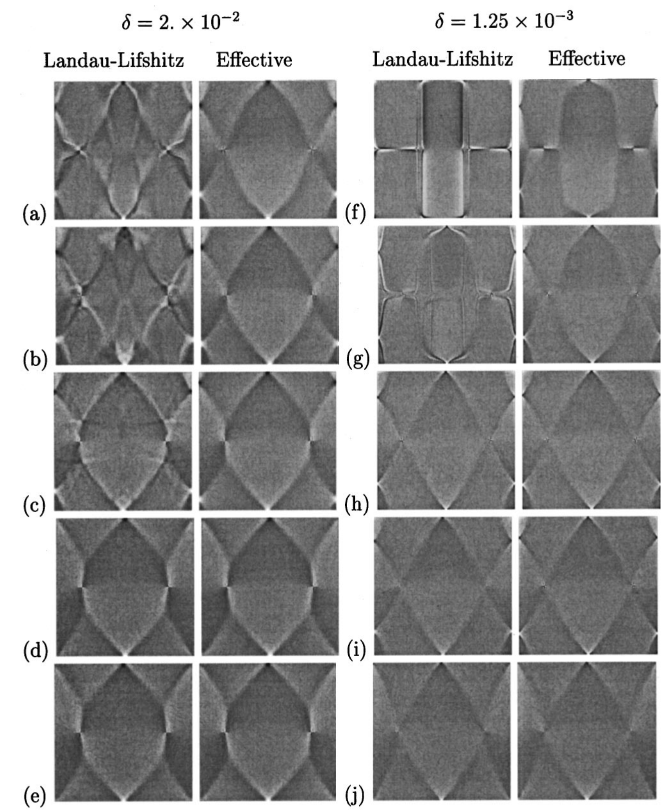

FIG. 2. Simulation of the full Landau–Lifshitz dynamics and the effective dynamics. The damping coefficient is fixed at $\alpha = 0.1$. The two columns on the left correspond to the Landau–Lifshitz dynamics and our effective dynamic equation for $\delta = 2\times 10^{-2}$. The frames correspond to the following times: $t=$ (a) 0.10, (b) 0.15, (c) 0.23, (d) 0.60, and (e) 1.0 ns. The two columns on the right correspond to $\delta = 1.25\times 10^{-3}$. The frames correspond to the following times: $t=$ (f) 0.10, (g) 0.25, (h) 0.50, (i) 1.0, and (j) 3.0 ns.

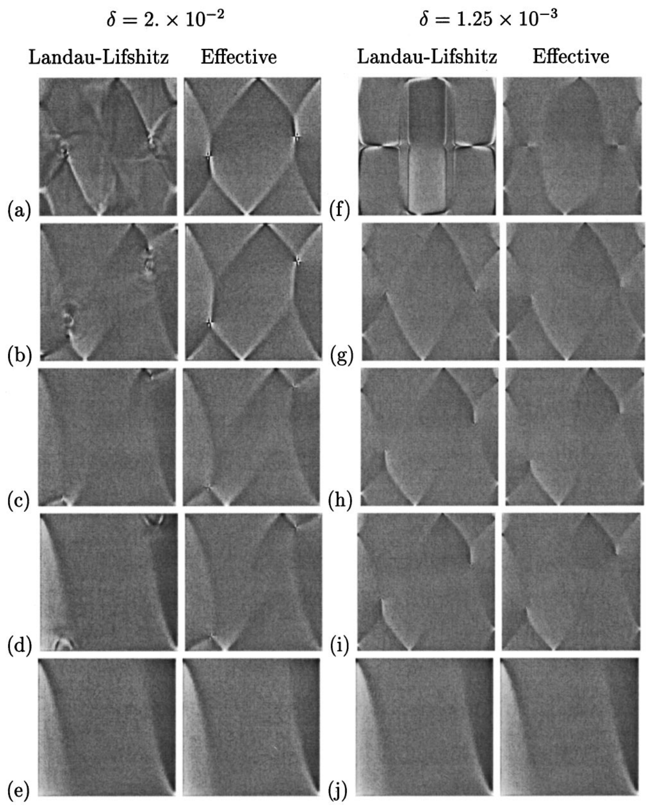

FIG. 3. Simulation of the full Landau–Lifshitz dynamics and the effective dynamics. The damping coefficient is fixed at $\alpha = 0.1$. The applied field is $\mathbf{h}_e = 2\delta[\cos(\pi/180), \sin(\pi/180)]$. The two columns on the left correspond to the Landau–Lifshitz dynamics and our effective dynamic equation for $\delta = 2\times 10^{-2}$. The frames correspond to the following times: $t=$ (a) 0.15, (b) 0.45, (c) 1.1, (d) 1.6, and (e) 1.8 ns. The two columns on the right correspond to $\delta = 1.25\times 10^{-3}$ for times: $t=$ (f) 0.15, (g) 0.65, (h) 1.75, (i) 2.10, and (j) 21.0 ns.

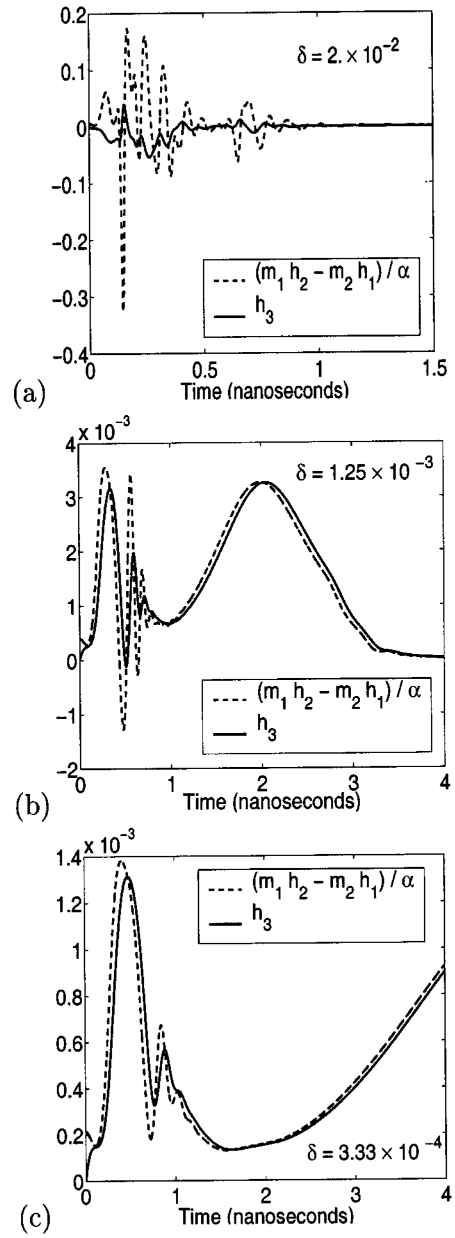

FIG. 4. Time evolution of h_3 vs $(m_2h_1 - m_1h_2)/\alpha$: $\delta=$ (a) 2×10^{-2}, (b) 1.25×10^{-3}, and (c) 3.33×10^{-4}.

what is expected: filter out the fast moving spin waves which are difficult to resolve numerically, but capture correctly the slower relaxational dynamics. The observation that spin waves are not important for the relaxational process might only be valid for the case of $\delta \ll \alpha$. In the opposite regime when $\delta \gg \alpha$, the opposite has been reported, namely, the spin waves are the mechanism for damping and relaxation.[14,15]

In our second experiment we have focused on the time history of one single magnetic spin in order to assess the validity of Eq. (15), that is,

$$h_3 \approx \frac{1}{\alpha}(m_1h_2 - m_2h_1), \quad \text{for } \alpha \gg \delta. \tag{20}$$

We have solved the full Landau–Lifshitz equation (1) for several values of the aspect ratio δ, fixing α at 0.1. In Fig. 4 we plot the time history of the two quantities, h_3, and $(m_2h_1 - m_1h_2)/\alpha$, at one grid location, for three different values of the aspect ratio. The picture on the top shows the results for aspect ratio $\delta = 2.\times 10^{-2}$, the picture in the middle shows the results for aspect ratio $\delta = 1.25\times 10^{-3}$, and the picture on the bottom shows the results for aspect ratio $\delta = 3.33\times 10^{-4}$. We see evidence of the presence of spin waves for all three different values of the aspect ratio. However, it is clear that the spin waves are heavily damped when the aspect ratio is reduced.

In our last experiment we have computed two hysteresis loops using the Landau–Lifshitz equation and our effective equation. The damping coefficient was fixed at $\alpha = 0.1$. Initially we apply a field $H_e = 2.5\,\delta M_s$ tilted 1° with respect to the OX axis. Once the magnetization reaches a steady state, we reduce the field, and repeat the operation until we reach the field $H_e = -2.5\,\delta M_s$. Then we repeat the operation, increasing the field, in order to close the loop. For our experiment we have used 200 different fields. Figure 5 shows the results for two different values of the aspect ratio. The hysteresis loops on the top correspond to the aspect ratio $\delta = 2.\times 10^{-2}$. The loops on the bottom correspond to the aspect ratio $\delta = 1.25\times 10^{-3}$.

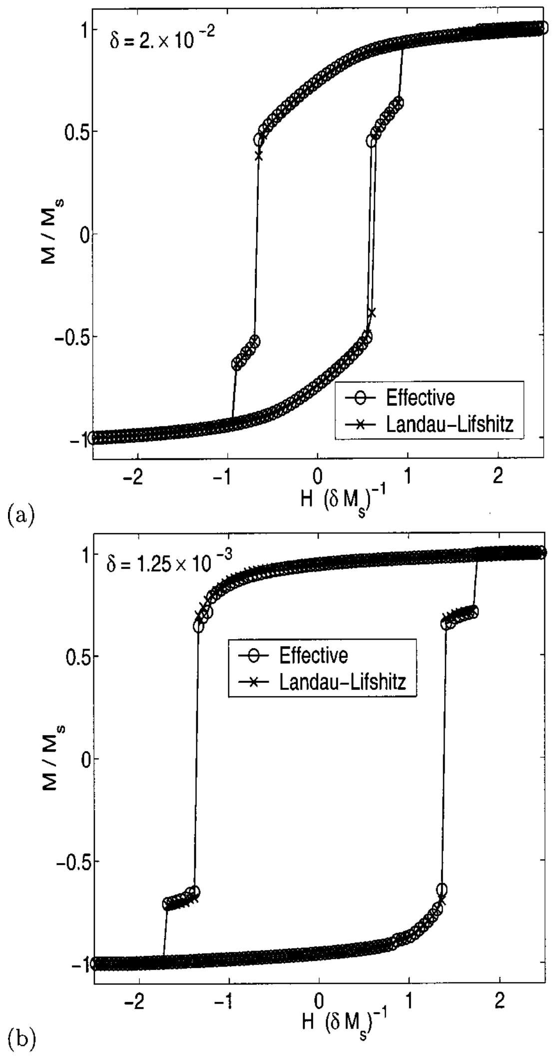

FIG. 5. Hysteresis loops obtained for each of the equations. (a) $\delta=2\times10^{-2}$ and (b) $\delta=1.25\times10^{-3}$.

We can see that the loops corresponding to the Landau–Lifshitz equation are almost identical to the loops obtained from the effective equation for the same value of the aspect ratio. From this experiment, we can see that the two equations have the same steady states, which again corroborates the previous statement that the effective dynamics filter out the spin waves, but capture the long term relaxation dynamics correctly.

CONCLUSION

We have systematically studied the effective dynamics in a ferromagnetic thin film. We found that when $\delta \ll \alpha$ the normal component is slaved to the in-plane components. Consequently the gyromagnetic term becomes a much more efficient damping term for the in-plane components. The spin waves are unimportant for the relaxational process of the magnetization field, and can be filtered out. The effective equation thus obtained is much easier to solve numerically. Our numerical results indicate that when $\delta \ll \alpha$, our effective equation gives an accurate description of the dynamics of the in-plane components. When δ is comparable to α, even though the detailed dynamics are not accurately modeled by the effective equation, the hysteresis loops can be calculated using the effective equation with satisfactory accuracy.

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[1] A. Hubert and R. Schäfer, *Magnetic Domains: The Analysis of Magnetic Microstructures* (Springer, Berlin, 1998).
[2] J. Daughton, Thin Solid Films **216**, 162 (1992).
[3] B. Heinrich and J. A. C. Bland, *Ultrathin Magnetic Structures I* (Springer, Berlin, 1994).
[4] B. Heinrich and J. A. C. Bland, *Ultrathin Magnetic Structures II* (Springer, Berlin, 1994).
[5] G. Prinz, Science **282**, 1660 (1998).
[6] W. F. Brown, Jr., *Micromagnetics*, Interscience Tracts on Physics and Astronomy (Wiley–Interscience, New York, 1963).
[7] L. Landau and E. Lifshitz, Phys. Z. Sowjetunion **8**, 153 (1935).
[8] C. J. García-Cervera, Ph.D. thesis, Courant Institute of Mathematical Sciences, New York University, New York, 1999.
[9] H. Riedel and A. Seeger, Phys. Status Solidi B **46**, 377 (1971).
[10] A. De Simone, R. V. Kohn, S. Müller, and F. Otto, Magnetic microstructures - a paradigm of multiscale problems, in Proceedings of the ICIAM, 1999.
[11] F. Otto, Lectures Notes at the Morningside Institute, Beijing, July 2000 (unpublished).
[12] Weinan E and X. P. Wang, SIAM (Soc. Ind. Appl. Math.) J. Numer. Anal. **38**, 1647 (2001).
[13] X. P. Wang, C. J. García-Cervera, and Weinan E, J. Comput. Phys. (in press).
[14] R. Koch (private communication).
[15] J. G. Zhu (private communication).