SOLUTION

1. (The car problem from last week's handout.) Let the speed of the cars be x mph. Since the two cars meet in the middle of the route, which is 120 miles long, we have

distance by A+distance by B=120.

Using the formula distance = speed×time, we get

$$\begin{aligned} (x)(2) + (x)(1) &= 120 \\ 3x &= 120 \\ x &= 40. \end{aligned}$$

Therefore, their speed is 40mph.

1. Let t be the number of hours after noon when they are 250 miles apart. If we draw a diagram for this problem, we will see that

distance by A + distance by B =250.

Using the formula distance = speed× times, we obtain

$$\begin{aligned} 60t + 40t &= 250 \\ 100t &= 250 \\ t &= 2.5. \end{aligned}$$

Therefore, they are 250 miles apart 2.5 hours after noon, i.e. at $2:30PM$.

3. First we find when Car A will arrive at San Francisco. Since time = distance / speed, we have

$$t = \frac{500}{60} = \frac{25}{3} = 8\frac{1}{3}.$$

Hence, Car A arrives at San Francisco $8\frac{1}{3}$ hours after $10AM$. Since Car B left San Diego at 3 hours after Car A, it would have traveled for $8\frac{1}{3} - 3 = 5\frac{1}{3}$ hours when Car A gets to San Francisco. So,

$$\text{distance traveled by B}=70 \times 5\frac{1}{3} = 70 \times \frac{16}{3} \approx 373 \text{ miles.}$$

This means Car B is still on the route when Car A is at San Francisco. Since Car B is traveling at a higher speed, if it passes Car A on the route, it would arrive at San Francsico before Car A. But our calculation showed that this does not happen. Therefore, Car B will not pass Car A before they arrive at San Francisco.

5. $f(x) = x^4 - x^2 + 2$: it is an even function so it is symmetric along the y-axis. (top left graph)

$g(x) = 30(4^{-x})$: this is an exponential graph with $-x$ in the exponent; so it should increase exponentially in the negtaive x-direction. (bottom right graph)

$h(x) = 4(x+1)(x-2)(x+3)$: this is zero exactly when $x = -1, 2$ and -3; so it crosses the x-axis at these points. (bottom left graph)

$k(x) = \frac{4}{2-x}$: the denominator is zero when $x = 2$; so it has a vertical aysmptote at $x = 2$. (top right graph)

8. Let V =volume of water that is in the pool, d =depth of water, and t =number of hours the water is pumped. Since the pool is in the shape of a cylinder, we have

$$V = \text{base area} \times \text{depth} = \pi r^2 \cdot d = 100\pi d.$$

Now, water is pumped in at a rate of $30000L/hr$. Converting it to m^3/hr, we get

$$30000\frac{L}{hr} \cdot \frac{1}{1000}\frac{m^3}{L} = 30\frac{m^3}{hr}.$$

Since the pool is initially empty, if the water has been pumped for t hours, the volume of water in the pool is

$$V = 30t \text{ (rate}\times\text{time)}.$$

Plugging this back to the equation we had earlier, we get

$$30t = 100\pi d.$$

Solving for d from this gives us

$$d = \frac{30t}{100\pi} = \frac{3t}{10\pi}.$$

9. You don't have to worry about this problem for the midterm. If you want to know how to do this just ask me.

10. We know that equation of a line has the form

$$y = mx + b$$

where m =slope and b =y−intercept. Let L be the line we are trying to find the equation of. Since L is parallel to $y = 2x - 4$, they would have the same slope, that is $m = 2$. Hence,

$$y = 2x + b.$$

To find what b is, since $(-1, 3)$ is a point of L, we can plug $x = -1$ and $y = 3$ into the equation. It gives us

$$\begin{aligned} 3 &= 2(-1) + b \\ b &= 5. \end{aligned}$$

Hence, the equation of L is $y = 2x + b$.

11. Again we start with

$$y = mx + b.$$

Since the line is perpendicular to $y - 4 = \frac{1}{3}(x - 3)$ (whose slope is $\frac{1}{3}$), the slope of our line is the negative reciprocal of $\frac{1}{3}$ and hence is -3. Thus,

$$y = -3x + b.$$

Since $(0, 0)$ lies on the line, we can plug in $y = 0 = x$, which gives us

$$\begin{aligned} 0 &= -3(0) + b \\ b &= 0. \end{aligned}$$

Hence, the equation of the line is $y = -3x$.

12. Using the information given, we know that

$$\begin{aligned} \text{John's house} &= (-3, -5) \\ \text{His school} &= (5, 1) \\ \text{Convenience store} &= (3, 0). \end{aligned}$$

To decide whether he will pass the convenience store if he walks in a straight line from his house to the school is the same thing as deciding whether the three points lie on the same line. We can check it by looking at their slopes.

$$\begin{aligned}\text{slope between house and school} &= \frac{-5-1}{-3-5} = \frac{-6}{-8} = \frac{3}{4}\\ \text{slope between school and convenience store} &= \frac{1-0}{5-3} = \frac{1}{2}.\end{aligned}$$

Since the slopes are not the same, it means they don't lie on the same line. Therefore, John will not pass the convenience store on his way.