

Application of Logarithms - Half life and Doubling

Suppose there is A_0 of a substance whose half life is K hours.

What is a formula that represents the amount of substance left after t hours?

Similarly, if there is A_0 of a substance initially and its doubling time is K hours.

What is a formula that represents the amount of substance left after t hours?

Note: As long as the units are consistent, you can change grams and hours to other units.

Problems.

1. At noon, you have 400g of a radioactive material that has a half-life of 3 hours. How much do you have at 3 : 00pm? How much do you have at 6 : 00pm? How much do you have at n hours after noon?

2. The half-life of element X is 50 years. If there are 80g initially

a) How much is there after 200 years?

b) How much is there after 17 years?

c) When will 40g remain?

d) When will 50g remain?

3. The level of radio-activity on the site of a nuclear explosion is decaying exponentially. The level measured in 1990 was found to be 0.7 times the level measured in 1980. What is the half life?

4. A population of rabbits doubles every year and there were 1 million rabbits on Jan 1, 1990.

a) How many rabbits are there in 1995?

b) When will there be 10 million rabbits?

Derivative and its Limit Definition

What is derivative? What are some of its interpretations?

Problems.

4. Let $f(x) = 3x + 4$. What is $f'(x)$?

5. Let $f(t) = 2t^2 + 3$. What is $f'(t)$? What is $f'(2)$?

6. Let $f(x) = \sqrt{x}$. What is $f'(x)$?

7. Below shows the graph of a function $y = f(x)$. You don't know what $f(x)$ is so you can't find what the derivative is. But looking from the graph, what can you say about $f'(1)$ and $f'(3)$?

a) Is $f'(1)$ positive or negative? Is $f'(3)$ positive or negative?

b) Is $|f'(1)|$ bigger or is $|f'(3)|$ bigger?

Average Velocity and Instantaneous Velocity (Or Rate of Change)

What are their geometric representations on the distance/height-time graph?

8. A car is traveling along a road and its position-time graph is given below. The position is measured in miles and time is measured in hour.

a) What is its average velocity in the time interval from $t = 2$ to $t = 4$?

b) Estimate its instantaneous velocity at $t = 3$.

9. The graph below shows the temperature in a city in a 24-hour period.

a) What is the average rate of change of temperature from 12pm to 4pm?

b) Estimate the instantaneous rate of change of temperature at 2pm.

c) Estimating from the graph, when is the temperature increasing at the highest rate? When is the temperature decreasing at the lowest rate?