# SOLUTION TO MIDTERM 3 REVIEW SHEET

Formula for half-life: $A(t) = A(\frac{1}{2})^{\frac{t}{K}}$.

Formula for doubling: $A(t) = A(2)^{\frac{t}{K}}$.

3. We can make $t = 0$ to when the year is 1980. Then, we know that the amount of radio-activity $t$ years after 1980 can be represented by the formula $A(t) = A(\frac{1}{2})^{\frac{t}{K}}$, where $A$ =amount in 1980 and $K$ =half life. We are given that

$$\text{level in1990} = 0.7(\text{level in 1980}).$$

This implies that

$$\begin{aligned} A(10) &= 0.7A(0) \\ A(\frac{1}{2})^{\frac{10}{K}} &= 0.7A \\ (\frac{1}{2})^{\frac{10}{K}} &= 0.7 \quad . \end{aligned}$$

Now take log on both sides to get

$$\begin{aligned} \frac{10}{K}\log(\frac{1}{2}) &= \log(0.7) \\ K &= \frac{10\log(1/2)}{\log(0.7)} \approx 19.4 \end{aligned}$$

Therefore, the half life is approximately 19.4 years.

4. a) We can make $t = 0$ to be when the year is 1990. This is a doubling problem with doubling time $K = 1$ year. So, the number of rabbits $t$ years after 1990 can be represented by the formula

$$A(t) = A(2)^{\frac{t}{K}} = A(2)^t.$$

Since we are told that there are 1 million rabbits in 1990, we can set $A = 1$ and make the unit of $A(t)$ to be million. This gives us

$$A(t) = 2^t.$$

The year 1995 corresponds to when $t = 5$ and $A(5) = 2^5 = 32$. Hence, there will be 32 millions rabbits in 1995.

b) We want to know that when there will be 10 million rabbits, i.e. $A(t) = 10$. This gives us the equation

$$2^t = 10 \implies t\log(2) = \log(10) \implies t = \frac{\log 10}{\log 2} \approx 3.3.$$

Hence, there will be 10 million rabbits sometime in 1993.

Interpretations of derivative: slope of tangent line, rate of change of $f(x)$

Limit definition of derivative:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$

6. Using the limit definition of derivative,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}.$$

The trick is to multiply both top and bottom by the conjugate of $\sqrt{x+h}-\sqrt{x}$.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} \\ &= \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})} \\ &= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})} \\ &= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}. \end{aligned}$$

Now we can plug in $h=0$ and it will give us

$$f'(x) = \frac{1}{2\sqrt{x}}.$$

7. a) $f'(1)$ is positive because the graph is increasing at $x=1$ (the slope of tangent at that point is positive). Similarly, $f'(3)$ is negative because the graph is decreasing at $x=3$ (the slope of tangent at that point is negative).

b) $|f'(3)|$ is bigger because the graph is steeper at $x=3$. It doesn't matter that $f'(3)$ is negative because we are taking the absolute value.

9. a) Average rate of change from $t=0$ to $t=4$ is

$$\frac{\text{change in temperature}}{\text{change in time}} = \frac{T(4)-T(0)}{4-0} = \frac{20-30}{4} = -\frac{10}{4} = -\frac{5}{2}.$$

Hence, the average rate of change is $-5/2°C/hr$.

b) There are many possible answers to this problem. Here we will estimate using the points $(0.5, 32)$ and $(1.5, 37)$. Then, taking the slope between them gives us

$$\frac{37-32}{1.5-0.5} = \frac{5}{1} = 5.$$

Hence, the instantenous rate of change of temperature at 1pm is approximately $5°C/hr$.

c) The temperature is increasing at the highest rate when the graph is going up and is the steepest. This occurs at about $t=1$. Similarly, the temperature is decreasing at the fastest rate when the graph is going down and is the steepest. This occurs at about $t=3$. You can have different estimations. As long as you have the same reasoning it will be an acceptable answer.