

MATH 34A REVIEW FOR THE FINAL (LOGS AND DERIVATIVES).

(i) Logarithm and applications.

1. Solve for x .

$$4^{x+1} = 5$$

2. Solve for x .

$$11 \cdot 6^{3x-1} = 2^{x+1}$$

3. The half-life of a substance is 2 hours. At noon, there are 20 grams of it.

a) How much is there at 2pm?

b) How much is there at 5 : 20pm? (Hint: What would t be at 5 : 20pm?)

c) When will 6 grams remain? (Use logs to solve for t .)

4. An element X is decaying exponentially. In 1990 it had a mass of 1000 grams. But now in 2012, its mass is measured to be 20 grams only. What is its half life?

5. A certain type of bacteria has been growing exponentially since 12AM. The doubling time is 2.5 hour.

a) How long does it take for the bacteria to triple? (Hint: You don't need A_0 to solve this part of the problem.)

b) If there are 1000 milligrams of it at 3AM, how much were there at 12AM?

6. Tim has \$50,000 in a bank that has an annual rate of 5% compounded yearly. How long will it take him to accumulate a sum of \$200,000 if he never withdraws or deposits extra money in the bank?

(ii) Derivatives and applications

1. Use this definition to find $f'(x)$ for each of the following functions.

a) $f(x) = 3x - 5$

b) $f(x) = x^2 + 1$

2. Let $f(x) = 3x^5 + 4x^2 - x + 10$.

a) Find the equation of the tangent line at $x = 1$.

b) Use a) to estimate $f(1.1)$. What is the percentage error?

3. Let $f(x) = x^2$. Find the equation of tangent line at $x = a$. If $(1, 1)$ is a point on this tangent line, what is the value of a ?

4. The price of a certain computer stock t days after it is issued for sale is $p(t) = 100 + 20t - 5t^2$ dollars. The price of the stock initially rises, but eventually begins to fall. During what period of time does the stock price rise? (Hint: Notice that time cannot be negative.) To make the most profit, when should you sell it?

5. A particular is moving along the x -axis and its position is given by the formula $x(t) = 3t^2 - t + 1$. What is the initial position of the particle? Over what time interval is the particle traveling towards the right?

6. A cylindrical metal can is to have no lid. It is to have volume $8\pi \text{ in}^3$. What height minimizes the amount of metal used (i.e. the surface area)?

7. What point on the graph $y = \sqrt{x}$ is closest to $(1, 0)$? (Hint: Instead of minimizing the distance minimize the square of the distance.)