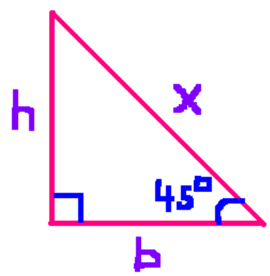
MATH 34A PROBLEM SOLVING SKILLS SOLUTIONS

It is possible that I made some mistakes when writing up these solutions. (Your brain doesn't function as well when you have to type math...) If you catch anything let me know.

I. General problems

1. A right-angle triangle has a 45° degree angle. If the area of the triangle is 25cm^2, what is the perimeter?

Solution. Let b be the base, h be the height, and x be the hypotenuse of the triangle. In order to find the perimeter, we need to find the lengths of all three sides.



Notice that the third angle is also 45° because the angles have to add up to 180°. So this is actually an isosceles triangle with $b = h$. Hence,

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(b)(b) = \frac{1}{2}b^2.$$

But we know that the area is 25. So,

$$\begin{aligned} \frac{1}{2}b^2 &= 25 \\ b^2 &= 50 \\ b &= \sqrt{50}. \end{aligned}$$

Now to find x, since it is a right-angle triangle, we can use the Pythagorean to get

$$\begin{aligned} b^2 + h^2 &= x^2 \\ b^2 + b^2 &= x^2 \\ 50 + 50 &= x^2 \\ 100 &= x^2 \\ x &= 10. \end{aligned}$$

Hence, the perimeter is

$$b + h + x = \sqrt{50} + \sqrt{50} + 10 = 2\sqrt{50} + 10 \text{ cm}.$$

2. There are 40 animals in a farmyard. Some are cows and some are chickens. In total there are 104 legs. How many chickens are there?

Solution. Let x be the number of chickens and y be the number of cows. Then,

$$\begin{aligned} \text{40 animals in a farmyard} \implies x + y &= 40 \\ \text{104 legs in total} \implies 2x + 4y &= 104. \end{aligned}$$

Now take the first equation and solve for y, we get $y = 40 - x$. Plugging it back into the second equation gives us

$$\begin{aligned} 2x + 4(40 - x) &= 104 \\ 2x + 160 - 4x &= 104 \\ -2x &= 104 - 160 \\ -2x &= -56 \\ x &= \frac{-56}{-2} \\ x &= 28. \end{aligned}$$

Therefore, there are 28 chickens in total.

II. Car problems

1. Car A leaves Sacramento at noon travelling at 60mph on a road 560 miles long to Los Angelos. Car B leaves Los Angelos at 2pm travelling at constant speed along the same road to Sacamento. They meet at 6pm. What was the speed of Car B? Furthermore, when they meet, are they closer to Sacramento or to Los Angelos?

Solution. Let the speed of B be $v$mph. Looking from the picture below,

we see that

$$\text{dist by A (grey part)} + \text{dist by B (blue part)} = 560.$$

Using the formula distance = speed $\times$ time, we get that

$$\begin{aligned} (60)(6) + (v)(4) &= 560 \\ 360 + 4v &= 560 \\ 4v &= 200 \\ v &= 50. \end{aligned}$$

Hence, the speed of B was 50mph.

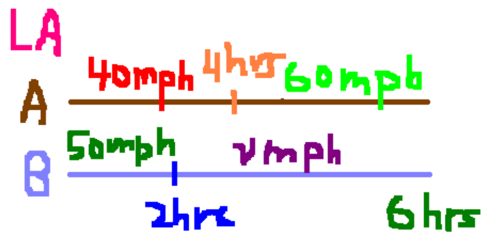
When they meet, as the calculation above shows,

$$\text{distance traveled by A} = 360\text{miles}.$$

Since the length of the route is 560miles, they must have closer to LA.

2. Car A and Car B leave Los Angeles at the same time and travel along the same route. Both cars arrive at the end of the route after 6 hours. Car A goes at 40mph for the first 4 hours then at 60mph for the rest of the time. Car B goes at 50mph for the first 2 hours and travels at a (different) constant speed for the remaining time. What is the speed with which car B travels the second part of the route?

Solution. Let $v$mph be the speed of B in the second part of the route.



First we can find the total distance of the route using information about A, using that distance = speed $\times$ time.

$$\begin{aligned}\text{Total distance} &= \text{distance of first part} + \text{distance of second part}\\ &= (40)(4) + (60)(2)\\ &= 160 + 120\\ &= 280.\end{aligned}$$

So this must be the distance B traveled also. Hence,

$$\begin{aligned}280 &= \text{distance traveled by B}\\ 280 &= (50)(2) + (v)(4)\\ 280 &= 100 + 4v\\ 180 &= 4v\\ v &= 45.\end{aligned}$$

Hence, the speed of B is the second part is 45mph.

III. Express in terms of problems

1. A rectangular field is to have an area of 2000m^2 and is to be surrounded by a fence. The cost of the fence is \$15 per meter of length. Express the total cost of the fence in terms of the width of the field.

Solution. Let w be the width, l be the length, and C be the total cost. Then,

$$\begin{aligned}C &= \text{perimeter} \times 15\\ &= (2w + 2l) \times 15.\end{aligned}$$

But we want it in terms of w so we don't want l. To get rid of l, we use the other information that is given.

$$\begin{aligned}\text{Area} &= 2000\\ wl &= 2000\\ l &= \frac{2000}{w}.\end{aligned}$$

Now plug it back into the total cost equation, we get

$$\begin{aligned}C &= (2w + 2\cdot\frac{2000}{w}) \times 15\\ &= 15(2w + \frac{4000}{w}).\end{aligned}$$

There are only numbers and w in this expression, so we are done.

2. Let C be a cylinder whose height is twice its radius. Express its surface area in terms of its volume.

Solution. Let h be the height, r be the radius, S be the surface area, and V be the volume. Then,

$$\begin{aligned} S &= \text{area of circles at top and bottom} + \text{vertical side} \\ &= \pi r^2 + \pi r^2 + 2\pi r h \\ &= 2\pi r^2 + (2\pi r)h. \end{aligned}$$

Note that the area of the vertical is just the area of a rectangle when you unfold the cylinder, and it has width = circumference of the circle and length=height. First we are given that $h = 2r$ we we can get rid of the h.

$$\begin{aligned} S &= 2\pi r^2 + (2\pi r)(2r) \\ &= 2\pi r^2 + 4\pi r^2 \\ &= 6\pi r^2. \end{aligned}$$

But we want S in terms of V so we don't want the r. Since

$$\begin{aligned} V &= \text{base area} \times \text{height} \\ V &= \pi r^2 \cdot h \\ V &= \pi r^2 \cdot 2r \\ V &= 2\pi r^2 \end{aligned}$$

and so

$$r = \sqrt{\frac{V}{2\pi}}.$$

So plugging this back into the equation for S, we get

$$\begin{aligned} S &= 6\pi(\sqrt{\frac{V}{2\pi}})^2 \\ &= 6\pi \cdot \frac{V}{2\pi} \\ &= 3V. \end{aligned}$$

And we are done because there are only numbers and V on the right hand side.

3. An acquarium with a square base has no top. There is a metal frame. Glass costs $\$3/m^2$ and the frame costs $\$2/m$. The volume is to be $20m^3$. Express the total cost in terms of the height.

Solution. Let s be the side length of the square base and h be the height. Let C be the total cost. Then,

$$\begin{aligned} C &= \text{cost for glass} + \text{cost for fram} \\ &= (\text{surface area})(3) + (\text{length of all sides})(2) \\ &= (s^2 + 4sh)(3) + (8s + 4h)(2) \\ &= 3(s^2 + 4sh) + 2(8s + 4h). \end{aligned}$$

Try to draw a picture to see how I got the surface area and length of all sides. But we need this in terms of h, so we need to get rid of s. But we know that

$$\text{Volume=base area} \times \text{height} = 20$$

so

$$\begin{aligned} s^2 h &= 20 \\ s^2 &= \frac{20}{h} \\ s &= \sqrt{\frac{20}{h}}. \end{aligned}$$

Plugging this back into the C equation, we get that

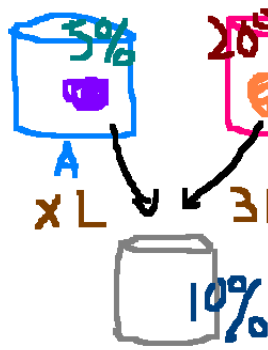
$$C = 3(\frac{20}{h} + 4\sqrt{\frac{20}{h}h}) + 2(8\sqrt{\frac{20}{h}} + 4h)$$

and we are done because there are only numbers and h in this expression.

VI. Mixing problems

1. Solution A contains 5% salt. Solution B contains 20% salt. How much solution A do you combine with 3 liters of solution B, to obtain a result containing 10% salt?

Let x liters be the amount of solution A.



So the amount of solution in the final mixture would be $x + 3$ liters. Now relating the amount of salt, we get that

$$\begin{aligned} \text{salt from A} + \text{salt from B} &= \text{salt in mixture} \\ (x)(5\%) + (3)(20\%) &= (x+3)(10\%). \end{aligned}$$

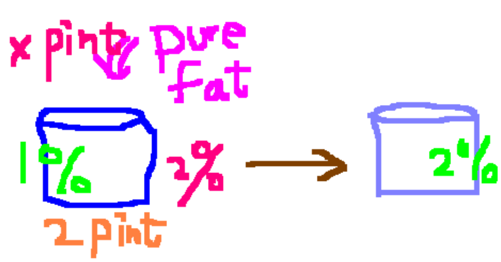
We can multiply the whole equation by 100 to get rid of the %. So,

$$\begin{aligned} 5x + 60 &= 10(x+3) \\ 5x + 60 &= 10x + 30 \\ 30 &= 5x \\ x &= 15. \end{aligned}$$

Hence, we need to add 15 liters from solution A.

2. I have 2 pints of milk that contains 1% fat. How much pure fat do I need to add in order to get milk that is 4% fat?

Solution. Let x pint be the amount of pure fat we need to add. Notice that pure fat means it is 100%.



So relating the amount of fat, we get

$$\begin{aligned} \text{amout of fat from original milk} + \text{amout of fat we added} &= \text{amout of fat in mixture} \\ (2)(1\%) + (x)(100\%) &= (x+2)(3\%). \end{aligned}$$

Again multiplying the whole equation by 100 to get rid of the %. So we get

$$\begin{aligned} 2+100x &= 3(x+2) \\ 2+100x &= 3x+6 \\ 97x &= 4 \\ x &= \frac{4}{97}. \end{aligned}$$

So we need to add $\frac{4}{97}$pint of pure fat.