

MATH 34A AVERAGE RATE OF CHANGE & DERIVATIVES

Derivative as a limit.

Some observations.

1. Units of the derivative
2. Derivative as rate of change

3. Signs of the derivative

Example.

3. Find the slope of tangent line of $y = 4x + 1$ at $x = 3$.

Understanding derivatives.

4. The population of a certain town t years after 2000 is given by the function $p(t)$. What do the following statements mean? Include units in your explanations.

a) $p(0) = 5000$

b) $p(10) = 20000$

c) $p'(5) = 1000$

d) $p'(6) = 0$

e) $p'(7)$ is negative.

Average velocity vs Instantaneous velocity.

5. The distance in miles of a car from its starting point after t hours is given by the formula $d(t) = t^2 + 3$.

a) What is the average velocity of the car during the time period $[1, 2]$? Units?

b) What is the average velocity of the car during the time period $[1, 1.5]$?

c) What is the average velocity of the car during the time period $[1, 1.1]$?

d) Based on your answers for parts a) to c), give an estimate of the instantaneous velocity of the car at $t = 1$.

e) Based on your answer for part d), is the car moving forward or backward at $t = 1$?

Average rate of change vs Instantaneous rate of change.

6. The total profit measured in millions of dollars that a company has made t years after 2000 is given by the function $p(t)$. We know that

$$p(5) = 25, p(6) = 20, p(7) = 18.$$

a) What is the average rate of change of the total profit from the year 2005 to 2007? Units?

b) What is the average rate of change of the total profit from the year 2005 to 2006?

c) Based on your answers in parts a) and b), do you expect the total profit to be more or less than 25 millions dollars in June 2005?