MATH 34A INTER/EXTRAPOLATION, EXPONENTS, LOGARITHM

I. Interpolation & Extrapolation

Idea: If you know two points on a graph, you can find the line joining them, and use it to approximate the function. Interpolation means you approximate something *between* them and extrapolation means you approximate something *beyond* them.

Example. The population in City A is 4 million in 1995 and 6 million in 2000.

i) Population (y-axis in million) is a function of time (x-axis in year). Which two points are given in the problem?

ii) Find the equation of the line joining the two points.

iii) Use ii) to approximate the population in 1998 (interpolation).

iv) Use ii) to approximate the population in 2004 (extrapolation).

II. Exponent and Logarithm

A. Exponential Laws

1. (multiplication→addition) $x^a x^b =$
2. (division→subtraction) $x^a/x^b =$
3. (exponent→multiplication) $(x^a)^b =$
4. (fraction→negative exponent) $\frac{1}{x^a} =$

The function log is the inverse of exp. The symbol

$$\log_b(x) \text{ represents the number } y \text{ that you need to raise } b \text{ to to get } x: \; b^y = x.$$

Use this definition to find:

1. $\log_{10}(1000) =$
2. $\log_2(\sqrt{2}) =$

3. $\log_3(\frac{1}{81}) =$
4. What is the *domain* of log , i.e. for what x-values is $\log(x)$ defined?

Since log is the inverse of exp, they cancel out each other:

$$10^{\log(x)} = x \text{ and } \log(10^x) = x.$$

The same works for any base, e.g. $e^{\ln(x)} = x$ and $\ln(e^x) = x$. Use this to simply:

1. $10^{\log(30)+\log(10)} =$
2. $e^{2\ln(3x)} =$
3. $\log_5(125) =$

The log laws are also in some sense opposite of the exp laws.

B. Logarithmic Laws
1. (addition→multiplication) $\log(a) + \log(b) =$
2. (subtraction→division) $\log(a) - \log(b) =$
3. (multiplication→exponent) $a\log(b) =$
4. (negative log→fraction) $-\log(a) =$

Use these laws to simplify: (e.g $\log(40) = \log(10 \cdot 4) = \log(10) + \log(2^2) = 1 + 2\log(2)$.)

1. $\log(800)$
2. $\log(9/20)$

C. Solving Exponential Equations

Idea: If the unknown is in the exponent, take log and then use law 3 to bring the power down.

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1. Solve for $x : 7^x = 5^{2x+1}$

a) First take log on both sides of the equation.

b) Then use law 3 to bring the powers down.

c) You should have a linear equation now. Isolate x.

2. Solve for $x : 2 \cdot 5^x = 3 \cdot 8^{x+1}$ (Hint: Other than law 3, which other law do you need here?)

III. Applications: Exponential Model

Some real-life problems (e.g. population growth) can be modeled using an exponential function.

A. Half-life

Example. The half-life of element X is 30 years. If there are 200g initially

a) How much is there after 30 years?

b) How much is there after 60 years?

c) How much is there after 90 years?

d) How much is there after n half-lives have passed (i.e. after $30n$ years)?

e) How much is there after 15 years? (Hint: How many half-lives are in 15 years?)

f) How much is there after 45 years?

g) How much is there after 49 years?

h) How much is there after t years?

Now try to generalize what you have just done. Suppose

$$\begin{aligned} K &= \text{the half-life of some substance (in years, say)} \\ A_0 &= \text{the initial amount.} \end{aligned}$$

Can you come up with a formula that tells you the amount $A(t)$ after t years?

B. Doubling Time

This is basically the same as half-life. If

$$\begin{aligned} K &= \text{the doubling time of some population (in years, say)} \\ P_0 &= \text{the initial amount.} \end{aligned}$$

What is a formula $P(t)$ that tells you the popoulation $P(t)$ after t years?

Use the formulae you came up with to solve the following problems.

1. A population of rabbits doubles every 2 years and there were 1 million rabbits on 1/1/1990.

a) Write down a formula representing the rabbit population t years after 1990.

a) How many rabbits are there in 1995?

b) When will there be 10 million rabbits?

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2. A type of bacteria has been growing exponentially since 12AM. The doubling time is 2 hours.

a) Suppose the initially amount is P_0 milligrams. Write down a formula $P(t)$ representing the amount of bacteria t hours after 12AM in terms of P_0.

b) How long does it take for the bacteria to triple? (Hint: You don't need to know P_0 for this.)

c) If there are 1000 milligrams of it at 3AM, how much were there at 12AM?

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3. The level of radio-activity on the site of a nuclear explosion is decaying exponentially. The level measured in 1990 was found to be 0.7 times the level measured in 1980. What is the half life?