

## Linear Approximation (Extra/Interpolation)

### Problems.

1. Consider  $f(x) = \sqrt[3]{x+1}$ . We know that  $\sqrt[3]{8} = 2$  and  $\sqrt[3]{27} = 3$ . Estimate what  $\sqrt[3]{15}$  is.
2. A city has population 250,000 in the year 2000 and 320,000 in the year 2010. Use these two points to estimate the population in 2005.

### Proportionality

$x$  is proportional to  $y$  means...

$x$  is inversely proportional to  $y$  means...

### Exponents and Logarithms

How does log work?

### Problems.

Use the definition of logarithm to find the following.

1.  $\log(1000000000) =$
2.  $\log\left(\frac{1}{100000}\right) =$
3.  $\log_5(25) =$
4.  $\log_2\left(\frac{1}{4}\right) =$
5.  $\log_3(3) =$

What are the five exponent-and-logarithm rules that you have learned?

	Exponent Rules	Logarithm Rules	
(1)	$10^a \times 10^b =$	$\log(xy) =$	
(2)	$10^0 =$	$\log(1) =$	
(3)	$10^{-a} =$	$\log\left(\frac{1}{x}\right) =$	
(4)	$(10^a)^p =$	$\log(x^p) =$	
(5)	$\frac{10^a}{10^b} =$	$\log\left(\frac{x}{y}\right) =$	

### Application of Logarithms

Solving equations

1.

$$3^x = 71$$

2.

$$2^{x+1} = 5^{2x}$$

3.

$$5 \cdot 7^x = 6^{-1+x}$$

### Half-life

Suppose we have  $A_0$  grams of a substance initially and we know that its half-life is  $K$  years. We can find a formula  $A(t)$  which tells us that amount of this substance after  $t$  years. What is it?

\*The half-life  $K$  can have units other than years. This formula still works as long as we make  $t$  to have the same units as  $K$ .

### Problems.

1. The half-life of element  $X$  is 50 years. If there are 80g initially
  - a) How much is there after 50 years?
  - b) How much is there after 100 years?
  - c) How much is there after 150 years?
  - d) How much is there after 1000000 years?
  - e) How much is there after  $x$  half lives have passed?
  - f) How much is there after  $t$  years?
  - g) How much is there after 17 years?
  - h) When will 50g remain?
2. There are 150g of element  $Y$  at noon. At 4 pm there are only 100g left. What is the half life of element  $Y$ ?

### Double-life

Suppose we have  $A_0$  grams of a substance initially and we know that its double-life is  $K$  years. We can find a formula  $A(t)$  which tells us that amount of this substance after  $t$  years. What is it?

### Compound Interest

Suppose we have  $M_0$  dollars in the bank initially and we know the interest rate is  $r\%$  compounded yearly. We can find a formula  $M(t)$  which tells us the amount of money after  $t$  years.

### Problems.

1. A bank pays 3% interest compounded annually. If you put \$5000 in your account now (Feb 2012), how much money will you have
  - a) in Feb 2013?
  - b) in Feb 2014?
  - c) in Feb 2015?
  - d)  $t$  years from now?
2. Same as above. Your goal is to have half a million in your bank account. How long is it going to take?