MATH 34B REVIEW ON CHAIN RULE

List of derivatives:

A. Basic derivatives:

1. $[x^n]' = nx^{n-1}$
2. $[e^x]' = e^x$
3. $[a^x]' = (\ln a)a^x$
4. $[\ln x]' = \frac{1}{x}$

B. Trigonometric functions:

1. $[\sin x]' = \cos x$
2. $[\cos x]' = -\sin x$
3. $[\tan x]' = \sec^2 x$
4. $[\csc x]' = -\csc x \cot x$
5. $[\sec x]' = \sec x \tan x$
6. $[\cot x]' = -\csc^2 x$

C. Inverse trignometric functions:

1. $[\arcsin x]' = \frac{1}{\sqrt{1-x^2}}$
2. $[\arccos x]' = \frac{-1}{\sqrt{1-x^2}}$
3. $[\arctan x]' = \frac{1}{1+x^2}$

Chain rule:

Formal statement: If $h(x) = f(g(x))$ and $u = g(x)$, then

$$\frac{dh}{dx} = \frac{df}{du} \cdot \frac{du}{dx} \text{ or } h'(x) = f'(g(x))g'(x).$$

Basically: It says

(derivative of a composite function)=(derivative of the outside function)(derivative of the inside function).

Examples:

1. $f(x) = (x^2 + 1)^9$

2. $f(x) = \cos(3x^5 + \frac{1}{x^3})$

3. $f(x) = e^{x^2+\sin(x)+2x}$

4. Show that $[2^x]' = (\ln 2)2^x$ using the chain rule and $[e^x]' = e^2$.

5. Show that $[\sec x]' = \sec x \tan x$ using the chain rule and $[\cos x]' = -\sin x$.

6. Derive the quotient rule

$$[\frac{f(x)}{g(x)}]' = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

from the chain rule and the product rule.

7. $f(x) = \frac{e^{3x}}{x^2-1}$

Additional practice: Find the derivatives of the following functions.

1. $f(x) = (3x+1)^2$
2. $f(x) = \sqrt{13x^2 - 5x + 8}$
3. $f(x) = (1 - 4x + 7x^5)^{30}$
4. $f(x) = (4x + x^{-5})^{\frac{1}{3}}$
5. $f(x) = (\frac{8x - x^6}{x^3})^{\frac{-4}{5}}$ (Hint: simplify before you differentiate.)
6. $f(x) = \sin(5x)$
7. $f(x) = e^{5x^2+7x-13}$
8. $f(x) = 2^{\cot x}$
9. $f(x) = 3\tan\sqrt{x}$
10. $f(x) = \ln(17 - x)$
11. $f(x) = \ln(4 + \cos x)$

(You might need to apply chain rule more than once for the problems below.)

12. $f(x) = \cos^2(x^3)$
13. $f(x) = \frac{1}{5}\sec^{-4}(4 + x^3)$
14. $f(x) = \ln(\cos^5(3x^4))$
15. $f(x) = \sqrt{\sin(7x + \ln(5x))}$

(You might need to apply the product rule on top of the chain rule for the problems below.)

16. $f(x) = \sqrt{\sin^2(2x) \cdot e^{2x}}$
17. $f(x) = e^{\sin(x^2)} \cdot \tan(4x^2 - 1)$

*18. $f(x) = \ln(e^{2x^2+1}(-x^2 + 10x + 2)^7)$ (Hint: simplify before you differentiate.)