WORHSHEET 2 SOLUTIONS

6. Derive the quotient rule

$$[\frac{f(x)}{g(x)}]' = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

from the chain rule and the product rule.

Write $\frac{f(x)}{g(x)} = f(x)g(x)^{-1}$. Then the product rule tells us that

$$[f(x)g(x)^{-1}] = [f(x)]'[g(x)^{-1}] + [f(x)][g(x)^{-1}]'.$$

Now apply the power rule and chain rule on $[g(x)^{-1}]'$, we get that

$$\begin{aligned} [f(x)g(x)^{-1}]' &= f'(x)g(x)^{-1} + f(x)[-g(x)^{-2} \cdot g'(x)] \\ &= \frac{f'(x)}{g(x)} + \frac{-f(x)g'(x)}{g(x)^2} \\ &= \frac{f'(x)g(x)}{g(x)^2} - \frac{f(x)g'(x)}{g(x)^2} \\ &= \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}. \end{aligned}$$

This proves the statement.

1. $f(x) = (3x+1)^2$

$$\begin{aligned} f'(x) &= 2(3x+1)^1 \cdot (3x+1)' \\ &= 2(3x+1) \cdot 3 \\ &= 6(3x+1) \end{aligned}$$

2. $f(x) = \sqrt{13x^2 - 5x + 8} = (13x^2 - 5x + 8)^{\frac{1}{2}}$

$$\begin{aligned} f'(x) &= \frac{1}{2}(13x^2 - 5x + 8)^{\frac{-1}{2}} \cdot (13x^2 - 5x + 8)' \\ &= \frac{1}{2}(13x^2 - 5x + 8)^{\frac{-1}{2}} \cdot (13 \cdot 2x - 5) \\ &= \frac{1}{2}(13x^2 - 5x + 8)^{\frac{-1}{2}}(26x - 5) \end{aligned}$$

3. $f(x) = (1 - 4x + 7x^5)^{30}$

$$\begin{aligned} f'(x) &= 30(1 - 4x + 7x^5) \cdot (1 - 4x + 7x^5)' \\ &= 30(1 - 4x + 7x^5) \cdot (0 - 5 + 7 \cdot 5x^4) \\ &= 30(1 - 4x + 7x^5)(-5 + 35x^4) \end{aligned}$$

4. $f(x) = (4x + x^{-5})^{\frac{1}{3}}$

$$\begin{aligned} f'(x) &= \frac{1}{3}(4x + x^{-5})^{\frac{-2}{3}} \cdot (4x + x^{-5})' \\ &= \frac{1}{3}(4x + x^{-5})^{\frac{-2}{3}}(4 - 5x^{-6}) \end{aligned}$$

5. $f(x) = (\frac{8x-x^6}{x^3})^{\frac{-4}{5}} = (8x^{-2} - x^3)^{\frac{-4}{5}}$

$$\begin{aligned} f'(x) &= \frac{-4}{5}(8x^{-2} - x^3)^{\frac{-4}{5}-1} \cdot (8x^{-2} - x^3)' \\ &= \frac{-4}{5}(8x^{-2} - x^3)^{\frac{-9}{5}} \cdot (8 \cdot -2x^{-3} - 3x^2) \\ &= \frac{-4}{5}(8x^{-2} - x^3)^{\frac{-9}{5}}(-16x^{-3} - 3x^2) \end{aligned}$$

6. $f(x) = \sin(5x)$

$$\begin{aligned} f'(x) &= \cos(5x) \cdot (5x)' \\ &= \cos(5x) \cdot 5 \\ &= 5\cos(5x) \end{aligned}$$

7. $f(x) = e^{5x^2+7x-13}$

$$\begin{aligned} f'(x) &= e^{5x^2+7x-13} \cdot (5x^2 + 7x - 13)' \\ &= e^{5x^2+7x-13} \cdot (5 \cdot 2x + 7) \\ &= e^{5x^2+7x-13}(10x + 7) \end{aligned}$$

8. $f(x) = 2^{\cot x}$

$$\begin{aligned} f'(x) &= (\ln 2)2^{\cot x} \cdot (\cot x)' \\ &= (\ln 2)2^{\cot x} \cdot (-\csc^2 x) \end{aligned}$$

9. $f(x) = 3\tan\sqrt{x}$

$$\begin{aligned} f'(x) &= 3(\sec^2\sqrt{x})(\sqrt{x})' \\ &= 3(\sec^2\sqrt{x})(x^{\frac{1}{2}})' \\ &= 3(\sec^2\sqrt{x})(\frac{1}{2}x^{-\frac{1}{2}}) \\ &= \frac{3}{2}(\sec^2\sqrt{x})x^{\frac{-1}{2}} \end{aligned}$$

10. $f(x) = \ln(17 - x)$

$$\begin{aligned} f'(x) &= \frac{1}{17-x} \cdot (17 - x)' \\ &= \frac{1}{17-x}(0 - 1) \\ &= \frac{-1}{17-x} \end{aligned}$$

11. $f(x) = \ln(4 + \cos x)$

$$\begin{aligned} f'(x) &= \frac{1}{4+\cos x} \cdot (4 + \cos x)' \\ &= \frac{1}{4+\cos x}(-\sin x) \\ &= \frac{-\sin x}{4+\cos x} \end{aligned}$$

(You might need to apply chain rule more than once for the problems below.)

12. $f(x) = \cos^2(x^3) = [\cos(x^3)]^2$

$$\begin{aligned} f'(x) &= 2\cos(x^3)\cdot(\cos(x^3))' \\ &= 2\cos(x^3)(-\sin(x^3))\cdot(x^3)' \\ &= 2\cos(x^3)(-\sin(x^3))(3x^2) \\ &= -6x^2\cos x^3 \sin x^3 \end{aligned}$$

13. $f(x) = \frac{1}{5}\sec^{-4}(4+x^3) = \frac{1}{5}[\sec(4+x^3)]^{-5}$

$$\begin{aligned} f'(x) &= \frac{1}{5}\cdot -5[\sec(4+x^3)]^{-6}\cdot(\sec(4+x^3))' \\ &= -\sec^{-6}(4+x^3)\cdot\sec(4+x^3)\tan(4+x^3)\cdot(4+x^3)' \\ &= -\sec^{-6}(4+x^3)\cdot\sec(4+x^3)\tan(4+x^3)\cdot 3x^2 \end{aligned}$$

14. $f(x) = \ln(\cos^5(3x^4))$

$$\begin{aligned} f'(x) &= \frac{1}{\cos^5(3x^4)}\cdot(\cos^5(3x^4))' \\ &= \frac{1}{\cos^5(3x^4)}\cdot([\cos(3x^4)]^5)' \\ &= \frac{1}{\cos^5(3x^4)}\cdot 5(\cos(3x^4))^4\cdot(\cos(3x^4))' \\ &= \frac{5\cos^4(3x^4)}{\cos^5(3x^4)}\cdot(-\sin(3x^4))\cdot(3x^4)' \\ &= \frac{-5\sin(3x^4)}{\cos(3x^4)}\cdot 3\cdot 4x^3 \\ &= \frac{-60x^3\sin(3x^4)}{\cos(3x^4)} \end{aligned}$$

A smarter way to do this: Write $f(x) = \ln([\cos(3x^4)]^5) = 5\ln(\cos(3x^4))$. Then,

$$\begin{aligned} f'(x) &= 5\cdot\frac{1}{\cos(3x^4)}\cdot(\cos(3x^4))' \\ &= \frac{5}{\cos(3x^4)}\cdot(-\sin(3x^4))\cdot(3x^4)' \\ &= \frac{-5\sin(3x^4)}{\cos(3x^4)}\cdot 3(4x^3) \\ &= \frac{-60x^3\sin(3x^4)}{\cos(3x^4)} \end{aligned}$$

15. $f(x) = \sqrt{\sin(7x+\ln(5x))} = (\sin(7x+\ln(5x)))^{\frac{1}{2}}$

$$\begin{aligned} f'(x) &= \frac{1}{2}(\sin(7x+\ln(5x)))^{\frac{-1}{2}}\cdot[\sin(7x+\ln(5x))]' \\ &= \frac{1}{2}(\sin(7x+\ln(5x)))^{\frac{-1}{2}}\cdot\cos(7x+\ln(5x))\cdot(7x+\ln 5x)' \\ &= \frac{1}{2}(\sin(7x+\ln(5x)))^{\frac{-1}{2}}\cdot\cos(7x+\ln(5x))\cdot(7+\frac{1}{5x}\cdot(5x)') \\ &= \frac{1}{2}(\sin(7x+\ln(5x)))^{\frac{-1}{2}}\cdot\cos(7x+\ln(5x))\cdot(7+\frac{1}{5x}\cdot 5) \\ &= \frac{1}{2}(\sin(7x+\ln(5x)))^{\frac{-1}{2}}\cdot\cos(7x+\ln(5x))\cdot(7+\frac{1}{x}) \end{aligned}$$

(You might need to apply the product rule on top of the chain rule for the problems below.)

16. $f(x) = \sqrt{\sin(2x) \cdot e^{2x}} = (\sin(2x)e^{2x})^{\frac{1}{2}}$

$$\begin{aligned}
f'(x) &= \frac{1}{2}(\sin(2x)e^{2x})^{\frac{-1}{2}} \cdot (\sin(2x) \cdot e^{2x})' \\
&= \frac{1}{2}(\sin(2x)e^{2x})^{\frac{-1}{2}} \cdot [\sin(2x) \cdot (e^{2x})' + e^{2x} \cdot (\sin(2x))'] \\
&= \frac{1}{2}(\sin(2x)e^{2x})^{\frac{-1}{2}} \cdot [\sin(2x) \cdot e^{2x} \cdot (2x)' + e^{2x} \cdot \cos(2x) \cdot (2x)'] \\
&= \frac{1}{2}(\sin(2x)e^{2x})^{\frac{-1}{2}} \cdot [\sin(2x) \cdot e^{2x} \cdot 2 + e^{2x} \cdot (\cos 2x) \cdot 2] \\
&= \frac{1}{2}(\sin(2x)e^{2x})^{\frac{-1}{2}} \cdot [2\sin(2x)e^{2x} + 2e^{2x}(\cos 2x)] \\
&= (\sin(2x)e^{2x})^{\frac{-1}{2}}[\sin(2x)e^{2x} + e^{2x}(\cos 2x)]
\end{aligned}$$

17. $f(x) = e^{\cos x} \cdot \tan(4x^2 - 1)$

$$\begin{aligned}
f'(x) &= e^{\cos x}(\tan(4x^2-1))' + \tan(4x^2-1)(e^{\cos x})' \\
&= e^{\cos x}(\sec^2(4x^2-1))(4x^2-1)' + \tan(4x^2-1) \cdot e^{\cos x}(\cos x)' \\
&= e^{\cos x}(\sec^2(4x^2-1))4 \cdot 2x + \tan(4x^2-1) \cdot e^{\cos x}(-\sin x) \\
&= 8xe^{\cos x}\sec^2(4x^2-1) - e^{\cos x}\sin x \tan(4x^2-1)
\end{aligned}$$

18. $f(x) = \ln(e^{2x^2+1}(-x^2+10x+2)^7)$

Using logarithm rules, we can rewrite

$$f(x) = \ln(e^{2x^2+1}) + \ln(-x^2+10x+2)^7 = 2x^2 + 1 + 7\ln(-x^2+10x+1).$$

Now $f(x)$ is easy to differentiate:

$$\begin{aligned}
f'(x) &= 2(2x) + 0 + 7(\frac{1}{-x^2+10x+1}) \cdot (-x^2+10x+1)' \\
&= 4x + \frac{7}{-x^2+10x+1}(-2x+10) \\
&= 4x + \frac{-14x+70}{-x^2+10x+1}
\end{aligned}$$