MATH 34B WORKSHEET 3

Meanings of derivative.

I. Applications of derivatives.

1. (*Ornithology*) An ornithologist determines that the body temperature of a certain bird species fluctuates over roughly a 17-hour period according to the cubic formula

$$T(t) = -68.07t^3 + 30.98t^2 + 12.52t + 37.1$$

for $0 \leq t \leq 0.713$, where T is the temperature in degrees Celcius measured t days from the beginning of a period.

a) Compute the interpret the derivative $T'(t)$.

b) At what rate is the temperature changing at the beginning of the period ($t = 0$) and at the end of the period ($t = 0.713$)? Is the temperature increasing or decreasing at the beginning and at the end of the period?

c) At what time is the temperature not changing? What is the bird's temperature at this time?

Additional Practice.

1. (*Bacterial growth*) The population P of a bacterial colony t days after observation begins is modeled by the quadratic function

$$P(t) = 103.5t^2 + 6900t + 230000.$$

a) Compute and interpret the derivative of $P'(t)$.
b) At what rate is the population changing after 1 day? Aftet 10 days?
c) What is the initial population of the colony?
d) How long does it take for the population to double?
e) At what rate is the population growing at the time it doubles?

II. Implicit differentiation.

1. $x^3 - y^2 = 5$

2. $xy^2 + \sin(y^2) = 3x$

3. $\frac{3x+y}{y^2} = x + 3y$.

Additional Practice.

1. $(x^2 + 3y^2)^5 = 2xy$
2. $\cos(e^y) + \cot(x^2) = 4$
3. $\ln(x^y) + \frac{1}{y} = \sec(x)$ (Hint: Use log rules to simplify $\ln(x^y)$ first.)
4. $e^{\sin y} + \ln(\frac{x^2}{y}) = 1$ (Hint: Use log rules to simplify $\ln(\frac{x^2}{y})$ first.)

III. Related rates

1. (*Demand rate*) When the price of a certain commodity is p dollars per unit, customers demand x hundred units of the commodity, where

$$x^2 + 3px + p^2 = 79.$$

How fast is the demand x changing with respect to time when the price is \$5 per unit and is decreasing at the rate of 30 cents per month?

2. (*Water pollution*) A circular oil slick spreads in such a way that its radius is increasing at the rate of 20ft/hr. How fast is the area of the slick changing when the radius is 200ft?

Additional Practice.

1. (*Supply rate*) When the price of a certain commodity is p dollars per unit, the manufacturer is willing to supply x hundred units, where

$$3p^2 - x = 12.$$

How fast is the supply changing when the price is \$4 per unit and is increasing at the rate of 87 cents per month?

2. (*Growth of a tumor*) A tumor is modeled as being roughly spherical, with radius R. If the radius of the tumor is currently $R = 0.54$ cm and is increasing at the rate of 0.13 cm per month, what is the corresponding rate of change of the volume of the tumor? (Hint: How do you find volume of a sphere?)