

MATH 34B DEFINITE INTEGRALS

Definite Integrals as Area

$$\int_a^b f(x) =$$

Examples. Find the following definite integrals by interpreting it as an area.

1. $\int_{-1}^5 3dx =$

2. $\int_0^2 xdx =$

Riemann sum approximation

Idea:

1. Estimate the following integral using left endpoint with $n = 4$.

$$\int_0^4 x^2 dx$$

2. Estimate the following integral using right endpoint with $n = 3$.

$$\int_1^7 \ln(4x) dx$$

3. Estimate the following integral using midpoint with $n = 4$.

$$\int_{-1}^1 \cos(x^3) dx$$

Fundamental theorem of Calculus

Part I.

$$f(x) = \int_c^x F(t) dt$$

is differentiable and $f'(x) = F(x)$.

Examples. Find the derivative of the following functions.

1. $g(x) = \int_1^x \sin(t) t^2 dt$

$$2. h(x) = \int_0^{x^2} \ln(t+3)dt$$

Part II.

$$\int_a^b f(x)dx = F(x) + C$$

Examples. Using FTC to find the following definite integrals.

$$1. \int_{-2}^3 e^{3x} dx =$$

$$2. \int_{-\pi}^{\frac{\pi}{2}} (2 \cos(x) + \sin(x)) dx =$$

$$3. \int_{-1}^2 \frac{x^2+2x}{x^3+3x^2+10} dx =$$

Some rules

Example. The functions $f(x)$ and $g(x)$ are continuous on the interval $-1 \leq x \leq 10$ and they satisfy

$$\int_{-1}^4 f(x)dx = 3, \quad \int_4^{10} f(x)dx = -5, \quad \int_{-1}^{10} g(x)dx = 8, \quad \int_4^{10} g(x)dx = -9.$$

Find

$$\int_{-1}^4 [2f(x) + 9g(x)]dx = \quad \text{and} \quad \int_{-1}^{10} [4g(x) - 3f(x)]dx =$$